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Public debt effects in theory-based models with an empirical assessment of the potential public debt

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Abstract: Using an endogenous growth model, we simulate a parametrized endogenous potential debt that a government could bear to finance its expenditure, derived from potential government spending. We run individual estimations for a sample of 20 advanced countries over the period of 1960–2015. The potential public debt is a debt limit curve that the economy could bear given its optimal production capacity, its economic growth and, its long-term interest rate. Compared to actual public debt, the potential debt drops below the actual one for many countries in times of crisis, especially after the 2008 financial crisis, suggesting that countries should reduce their debt to keep it away below their potential debt. The simulated potential public debt is sensitive to the interest rate and economic growth gap rather than the elasticity of public capital. For many countries, potential debt decreases faster in response to a rapid accumulation of actual debt, leading to an intersection between the two curves that generally materializes in times of crisis (Portugal, Spain, France, Italy, Greece, Belgium and the United Kingdom). For other countries, the debt situation is safer, as potential debt is higher than the actual debt moving in parallel with it (Australia, Denmark, New Zealand, Norway, Sweden and Switzerland). Besides, the public capital stock elasticities differ across countries, with an average of around 0.3 and the potential public expenditure series shows a decreasing trend over time, but it is still higher than the one observed in many countries in the sample. This is driven by the decreasing trend of public capital stock productivity, generally observed in the Great Moderation Era (1985–2015). Therefore, to increase potential public expenditure, and hence the potential public debt, governments should enhance such productivity. This might be achievable through the choice of higher productive public capital, which implies selected public projects with higher multipliers.

JEL Classification: B22, B23, C51, H54, H63.

Keywords: Potential Debt, Infinitely Lived Agents, Endogenous Growth, Government Expenditure, New Keynesian Models, Public Capital.

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Les effets de la dette publique dans les modèles théoriques

avec une évaluation empirique de la dette publique potentielle

Résumé : À l'aide d'un modèle de croissance endogène, nous simulons une dette potentielle endogène, dérivée des dépenses publiques potentielles, qu'un gouvernement pourrait supporter pour financer ses dépenses. Nous effectuons des estimations individuelles pour un échantillon de 20 pays avancés sur la période 1960-2015. La dette publique potentielle est une courbe de dette limite que l'économie pourrait supporter étant donnés sa capacité de production optimale, sa croissance économique, et son taux d'intérêt à long terme. Pour de nombreux pays en temps de crise, en particulier après la crise de 2008, la courbe de la dette potentielle passe en dessous de celle de la dette observée suggérant ainsi pour ces pays la nécessité de réduire leur dette afin de la maintenir en dessous de la dette potentielle. La dette publique potentielle simulée est sensible à l'écart entre le taux d'intérêt et la croissance économique, plutôt qu'à l'élasticité du capital public. Pour de nombreux pays, la dette potentielle diminue plus rapidement en réponse à une accumulation rapide de dette, conduisant à une intersection entre les deux courbes concrétisées notamment en temps de crise (Portugal, Espagne, France, Italie, Grèce, Belgique et Royaume-Uni). Pour d'autres pays, la situation de la dette est plus rassurante, la dette potentielle demeurant plus élevée que la dette observée et évoluant parallèlement à celle-ci (Australie, Danemark, Nouvelle-Zélande, Norvège, Suède et Suisse). En outre, les élasticités du stock de capital public diffèrent d'un pays à l'autre, enregistrant une moyenne d'environ 0,3. La série des dépenses publiques potentielles montre une tendance à la baisse, tout en restant supérieure à celle observée dans de nombreux pays de l'échantillon. Cela est dû à la tendance à la baisse de la productivité du stock de capital public, généralement observée durant l'époque de la grande modération (1985-2015). Par conséquent, pour augmenter les dépenses publiques potentielles, et donc la dette publique potentielle, les gouvernements devraient améliorer cette productivité. Cela pourrait se réaliser grâce au choix d'un capital public plus productif, ce qui implique la sélection de projets publics avec des multiplicateurs plus élevés.

Classification JEL : B22, B23, C51, H54, H63.

Mots-clés : Dette potentielle, Agents à durée de vie infinie, Croissance endogène, Dépenses publiques, Modèles nouveaux keynésiens, Capital public.

1. Introduction

To assess the effects of government debt, many theoretical growth models have been designed. The first class of these models is the infinitely lived agents' models, initiated by Ramsey (1928). Later, the debate between economists about agents' lifetime horizon and the type of intergenerational operative transfer linkages between such agents led to building the concept of overlapping generations' (families/dynasties) models to account for the existence or absence of continuity between generations provided by such linkages (Samuelson, 1958; Diamond, 1965). In particular, the most debated ideas are the degree of implication of altruism between generations (such as a motive bequest) and the lifetime agents' horizon in determining the existence of government debt effects on agents' behaviour in terms of saving, capital accumulation, consumers' utility and interest rate. Important contributions to this field are those of Diamond (1965), Barro (1974), Blanchard (1985), Buiter (1988), Aiyagari (1985, 1989) and Weil (1989).[2]

In this regard, two major propositions emerged. The first, according to the neoclassical framework, is that government debt crowds out private capital by increasing interest rates (Modigliani, 1961). The second is the Ricardian equivalence for which its advocates show that debt neutrality could happen depending particularly on the existence of operative altruistic links (bequests) between generations (Barro, 1974). In this way, the debate in the economic literature emerged especially in the 1970s and 1980s, with useful contributions modelling fiscal policy insights, precisely the debt–tax swap and its effects on welfare utility and interest rates.

While the most important property characterizing the majority of neoclassical models considering government debt is that they build their reasoning on household behaviour towards public expenditure and government debt, other literature on the political economy of debt assesses the effects of public debt, studying the behaviour of governments and the influence of economic and political institutions. In this regard, two approaches are debated. The first is the normative approach, where the government is considered a social planner (a benevolent social planner), for whom the priority is to maximize the social welfare of its individuals in society (Barro, 1979; Lucas and Stokey, 1983; Aiyagari et al., 2002). The second is the positive approach, considering public debt as a state variable used by each government as a strategy to influence its successor's choices or as a way to shape private economic agents' expectations (Persson and Svensson, 1989; Alesina and Tabellini, 1990). In this regard, the most disseminated ideas are related to the effects of fiscal policy (government debt and spending policies) under governments following committed rules versus discretionary policies. In particular, the government time inconsistent[3] actions have an impact on the way economic agents form their expectations, which affects their economic decisions.

[2] See, for example, De la Croix and Michel (2010) and Weil (2008) for a large literature review.
[3] The government time inconsistency issue was raised by Kydland and Prescott (1977).

The public debt effects are also assessed theoretically and empirically in the class of endogenous growth models. The pioneering contributions in this modelling area flourished with the development of endogenous growth models, especially by Romer (1986) and Lucas (1988). These models were brought as an alternative to the neoclassical growth model of Solow (1956) and Swan (1956) (the Solow–Swan model). The principal characteristics of such models are focusing on the accumulation of knowledge and its endogenization (whether this is embodied in the form of technological progress or in the form of human capital). Since then, many sources of growth have been integrated, particularly to the production function, as inputs such as innovation, human capital, ideas and government goods, for example (Jones, 2003, 2005, 2019; Jones and Romer, 2010; Bloom et al., 2019). The latter and similar contributions assess the effects of fiscal policy (taxes, government debt and spending) generally integrating the public sector into the productive sector (Barro, 1990). However, high government spending (especially unproductive spending), jointly with assumed distortionary taxes, leads to low per-capita growth rates, according to the neoclassical growth theory, or to lower growth, according to endogenous growth theory. These results contrast with the growing empirical evidence that higher government spending and taxes (relative to the size of the economy) are not negatively correlated with the growth rate (Corsetti and Roubini, 1996).[4]

Recently, fiscal policy, and particularly government debt, was also modelled under the class of new Keynesian models,[5] despite these models still actively prioritizing monetary policy analysis (see, for example, Rupert and Šustek, 2019). These models differ from the overlapping generations models in many aspects. For example, instead of considering all taxes as a lump sum, as assumed in the overlapping generations models, recent new Keynesian models have considered fiscal policy assuming distortionary taxes. Furthermore, they join (intersect with) the literature of the political economy of debt by considering the scope of government actions and discuss government policies under commitment or discretion rules. In particular, some authors argue that optimal public debt would follow a random walk process whenever the government can achieve a time-inconsistent policy commitment (Benigno and Woodford, 2003; Schmitt-Grohe and Uribe, 2004).

This paper is structured as follows. Section 2 reviews the existing literature, namely, the government debt effects in economic growth models, as debated in the old generation models and the endogenous growth models, as well as the very recent new Keynesian models and in the new political economy of debt (positive approach).[6] Section 3 describes the theoretical framework used to assess the effects of government debt (government expenditure) on the economy. Section 4 describes the empirical evidence. Section 5 concludes.

[4] Corsetti and Roubini (1996) interpret such facts by the imprecise distinction between productive and non-productive public spending. Many forms of public spending affect the productivity of the economy differently, either directly or indirectly. The theoretical prediction of a negative tax rate effect on growth is weakened once public spending can be qualified as productive.
[5] See, for example, Leith and von Thadden (2008), Leith and Wren-Lewis (2013) and Rossi (2014).
[6] These models are categorized, and their main results summarized, in Table A.1 of the appendix.

2. Literature review

This section is dedicated to the assessment of government debt effects in the theoretical models. Without claiming completeness, we will make an inventory of the most important contributions, modelling the effect of debt on the behaviour of economic agents and the subsequent consequences on the macroeconomic aggregates. First, we consider the government effects in the infinitely lived agents and the overlapping generations models (henceforward, ILA and OLG models). Second, a summary of the public debt effects in endogenous growth models is presented. Third, the effects of public debt are also considered and discussed in the economic literature of the new political economy of fiscal policy, also known as the positive approach of public debt and fiscal policy. We also present some studies assessing the effects of public spending and debt in the recent class of new Keynesian models, specifically in an integrated framework of fiscal and monetary policy. The fourth section concludes with two critiques of Mankiw (2000) for the ILA and OLG models and the Chari et al. (2009) critique of the new Keynesian models.

2.1. Government debt effects in the ILA and OLG models

The main ideas debated in the ILA and OLG models are mostly related to the way government bonds affect the steady state equilibrium interest rate (hence, capital accumulation) and consumption (welfare utility). In this way, two major ideas are contrasted. The proposition that public debt increases equilibrium interest rates in the steady state was confronted by the debt-neutrality idea known as Ricardian equivalence.[7] The first idea, as illustrated by Modigliani (1961), is that, in a full employment model, increasing government debt increases the conception of households' net wealth, which raises consumption and hence reduces saving, resulting in an increase in real interest rates. This reduces the output share resulting from the accumulation of capital. However, for Barro (1974), the idea that the "government debt effect on aggregate demand depends on the assumed increase of the households' net wealth" is only true in the non-full employment framework. In a full employment context, public debt effects could have no wealth effect if agents (generations) were economically connected by operative intergenerational transfers.

In this regard, two slightly different versions of the same neoclassical core model should be distinguished. The first class of model of growth assumes infinitely lived agents (ILA), (Ramsey, 1928; Cass, 1965). The second version is OLG models, which instead, have shifted the debate, considering the intergenerational linkages between generations rather than their lifetime horizon[8]

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[7] Buiter (1988) defines Ricardian equivalence by stating that: "*There is debt neutrality if, given a program for public expenditure on current goods and services over time, the real equilibrium of the economy is not affected by a change in the pattern over time of lump-sum taxes. If there is debt neutrality, for instance, the substitution of borrowing today for lump-sum taxation today…does not affect the current and future behavior of private consumption and capital formation.*"

[8] According to some authors (Barro, 1974; Weil, 1989), having operative linkages in OLG models between some economic agents with defined finite time horizons leads to infinite lifetime connected generations in the OLG models.

(Samuelson, 1958; Diamond, 1965). The subsequent contributions based on these models show that the debt effect is due to the degree of altruism existing between young and old generations rather than the presence of the infinitely lived agents (linkages that could result in agents with infinite lifetime horizons). This created an intense debate between the neoclassical and proponents of the Ricardian-equivalence conjecture (Barro, 1976; Feldstein, 1976).

Thereby, Diamond (1965) constructed an OLG model to study the effects of government debt (domestic and foreign) on the long-term competitive equilibrium. The economy assumes an infinitely long life and agents living for two periods, working in the first and retiring in the second. The model particularly assesses the effects of domestic and foreign debt on both utility level and the equilibrium interest rate. In this model several key assumptions are made about national debt. First, governments have a one-period maturity avoiding the issue of the expected capital returns. Second, the debt pays the current interest rate. Third, the debt–labour ratio is held constant. Fourth, taxes are assumed as a lump sum on the youth generation.

In this model the effects of government debt on utility and equilibrium interest rates depend on the coexistence of external and internal debt in the portfolio of government debt. Internal debt raises the interest rate and decreases the utility level in the efficient competitive equilibrium, and may increase or decrease it in the inefficient equilibrium. In particular, in the absence of external debt, domestic debt may increase utility in the case of inefficient competitive equilibrium. External debt increases the gap between the equilibrium interest rate and economic growth. Specifically, it moves the interest rate away from the golden rule solution,[9] which, in turn, reduces utility. In the case of an efficient competitive solution, external debt reduces the utility level of individuals in the long-term equilibrium. However, in the case of an inefficient solution, the external debt effect can raise or lower the utility independently of the existence of internal debt.[10] Furthermore, the debt swap (the substitution of internal debt by external debt) positively influences the interest rate and hence negatively influences utility in the efficient equilibrium, while it could reduce or increase it in the inefficient case. The author differentiates between four effects of public debt on utility: the effect of domestic debt following changes in the taxes required to finance it, the debt effect in the relative factor payments, the effect of external debt, and the debt swap effect.

Nevertheless, Barro (1974) constructed a model of overlapping generations based on Samuelson–Diamond's core model (Samuelson, 1958; Diamond, 1965) to argue that government bonds displace the interest rate, and utility in the steady-state equilibrium does not necessarily hold and depends, in particular, on the existence of operative altruistic links (bequests) between generations. Expressly, in the absence of such linkages, the current living generation does not necessarily consider the welfare of its dependants (future generations). Therefore, shifting the tax burden to

[9] According to Phelps (1961): "*A golden age means a dynamic equilibrium in which output and capital grow exponentially at the same rate so that the capital–output ratio is stationary over time.*"

[10] The principal findings of the Diamond model, for which government deficits raise long-term interest rates, have been empirically tested in many papers, with a mixture of numerical results (Ni, 1999).

the future may change the effective lifetime budget of the current living generation. Alternatively, operative altruistic links may cancel the effect on the lifetime budget of the current living generation, as the latter is aware of the welfare of the future generation. Consequently, debt neutrality is guaranteed, as the debt-for-tax swap does not affect the resource allocations and interest rates of the current generation in this case. For Cukierman and Meltzer (1989), it is sufficient that the bequest motives do not operate for some economic agents only, to conclude under majority rule, for the non-neutrality of government debt.

Unlike Diamond's (1965) model, which considers the infinite lifetime horizon, Blanchard (1985) studies the effects of debt and accumulation of deficit in a finite lived agent's horizon.[11] This is conducted by constructing what he called an "index of fiscal policy" supposed to capture the effects of current and anticipated fiscal policy. This index has two parts, of which one shows the effect of both changes and levels of government spending on aggregate demand, while the second highlights the effects of government finance; this is reflected by the effects of both government debt and the expected sequence of accumulated deficits on aggregate demand.

The main conclusions are that a government debt increase displaces the steady-state level of foreign assets in agents' wealth in an open economy[12] and decreases the steady-state level of capital and consumption in a closed economy. Consequently, the government can choose any level of steady state of consumption (open economy) and capital (closed economy) by simply choosing its level of debt. Similarly, a decrease in current lump-sum taxes increases human wealth and consumption. The longer the taxes are differed (i.e. shifted to future generations), the larger the effect. In summary, the increase in government debt and taxes creates initial wealth effects on consumption, leading to capital decumulation, which makes capital and consumption lower in the new steady-state level.

For other authors, the way the tax cut is distributed among economic agents plays an important role in determining the effects of fiscal policy. In this way, Aiyagari (1985) used a modified version of the OLG model based on Samuelson (1958) to show that Ricardian equivalence[13] depends on the way the tax cut is distributed among agents. In particular, debt neutrality holds in an OLG framework where this distribution does not change agents' wealth allocations. The choice of the OLG model (instead of an ILA model) was attributed to the fact that this type of model allows for a changeable real interest rate (which may be above or below the real growth rate) to different deficit policy regimes. Furthermore, the OLG models consider heterogeneous agents, which enable

[11] Blanchard (1985) imposes a constant instantaneous probability of death over agents' lifetime. This makes different agents with different ages and different levels of wealth have the same horizon and assumes the same propensity to consume. This easily enables the aggregation of the consumption function. Diamond's (1965) model adopts, however, specified population and age structures to avoid the aggregation issue.

[12] The larger the horizon of agents, the smaller the effect. Specifically, in an infinite horizon, the level of government debt has no effect on the steady-state level of foreign assets (Blanchard, 1985).

[13] Ricardian equivalence is summarized as "the debt–tax swap for financing government spending, [which] has no effect on the interest rates and consumption allocations".

taxes to be distributed differently among them. On the contrary, ILA models do not allow the real interest rate to go below economic growth, while, in addition, they assume identical economic agents yielding a uniform tax distribution.

Aiyagari's results were addressed in response to Miller and Sargent (1984), for whom "a shift to a different regime with permanently higher deficits will raise the interest rate and may make it exceed the growth rate". Aiyagari (1985) shows that this statement depends on how wealth is distributed, and this does not hold when the distribution of wealth is kept unchanged. His model shows that a higher level of government spending (or similarly a cut in total taxes) can be financed by debt alone at an unchanged (and negative) interest rate and with unchanged total taxes, if the taxes are distributed in a way that maintains wealth distribution. According to the author, this implies reducing taxes for the younger generation as savers and increasing taxes on the older generation of non-savers, while keeping total taxes constant. Thus, any actual effect of higher government spending on interest rates may arise because distributional impacts are not being controlled for, and not simply because the deficit is higher. So, for Aiyagari (1985), the validity of Ricardian debt neutrality depends on the way taxes are distributed among taxpayers and not on the length of the lifetime horizon.

Considering the operative linkages between agents, other contributions consider instead that the effects of government bonds on economic agents' wealth are related to the way the tax bases are foreseen. In this regard, Buiter (1988) builds on the Yaari-Blanchard version[14] of the overlapping generations model to show that the real equilibrium of the economy (private consumption, capital and relative prices) is independent of the pattern of government debt and lump-sum taxation over time. For the author, the difference between the expected government tax base and the future tax base of individuals that are alive today explains the variations over time in the pattern of lump-sum taxation. The former tax base represents the resources of individuals alive today, and those yet to be born, while the latter represents the resources owned by individuals alive today only, and not the resources of individuals yet to be born. In particular, economic agents that are not linked to their future generations through intergenerational transfers do not integrate the resources of these successors' generations into their inter-temporal budget constraint. In this regard, Buiter (1988) shows that debt neutrality holds if, and only if, the population growth rate and the individual probability of death equal zero.[15] Furthermore, under the latter condition, a non-zero labour productivity rate will not destroy this debt neutrality.

[14] Private consumption behaviour is modelled following the Yaari-Blanchard approach (Yaari, 1965; Blanchard, 1985).

[15] Blanchard (1985) considers that only an uncertain lifetime condition (a positive non-zero probability of death) is sufficient to invalidate the debt-neutrality conjecture.

Contrary to the models that take into account intergenerational transfers and those considering the infinite lifetime horizon induced by such linkages as prerequisites for debt-neutrality validation,[16] Weil (1989) developed a model of "overlapping families of infinitely lived agents"[17] to show that the assumption of the "infinite lifetime horizon"[18] is not necessarily a condition that induces Ricardian debt neutrality, as well as, inversely, finite horizons not necessarily implying the violation of Ricardian neutrality. The model assumes new cohorts entering the economy over time, for which operative intergenerational linkages between some, but not all, agents, result in "*partial linkages sufficient to endow any agent alive at any date with an effectively infinite economic horizon*". For Weil (1989), newly arriving families are not linked to pre-existing dynasties through operative intergenerational transfers.[19] The infinite lifetime of the agents (dynasties) is guaranteed by the continuous arrival of families who are not linked by operative intergenerational transfers (no intergenerational altruism). The number of newly arriving families (cohorts), which measures economic disconnectedness (and heterogeneity of the population), is also the growth rate of the population. The model is viewed as an extreme version of Blanchard's (1985) version, as it focuses on the birth rate of new arrival families while setting the probability of death to zero – "*agents are born but never die*". The model of Blanchard (1985) assumes a constant population by equalling the birth to death rates. Buitter's (1988) model, however, based on Yaari (1965), assumes distinct birth and death rates.

By introducing government bonds in his model, which involves levying lump-sum taxes, Weil (1989) shows that the equilibrium interest rate hinges on the government financing decision and the rate (speed) of newly arriving cohorts (families). In particular, government bonds increase the equilibrium interest rate for a positive non-zero population growth rate. For the author, the anticipated taxes to pay the issued public debt are expected to be compensated by generations that have not yet been born. However, generations that are alive today do not consider these taxes in their consumption. This makes them better off and pushes them to spend more, "*not because they might not be alive when future taxes are levied (they will, as they live forever), but because the future tax base will include new agents to whom they are not economically connected. The real interest rate must hence rise to maintain aggregate consumption at its market-clearing level. Infinite lifetimes are therefore not inconsistent with the violation of the Ricardian debt neutrality*

[16] The relationship between altruistic intergenerational transfers, infinite time horizons and Ricardian debt neutrality has been widely debated: "*It is widely argued that operative intergenerational transfers between all generations, because they imply infinite horizons, lead to Ricardian neutrality (cf. the debate between Feldstein (1976) and Barro (1976)). It is also suggested that finite lifespans lead to the violation of the Ricardian proposition (Blanchard, 1985)*" (Weil, 1989).

[17] The assumption of "finite lifetime" adopted by OLG models is considered useless in this model by the author.

[18] The effective length of consumers' planning horizon is infinite.

[19] "*Consider, for instance, a primogeniture economy in which a parent only loves his first-born heir, enough to leave him a bequest. Assume that all parents have children, and that children do not love their parents. Each child, whether first-born or not, is, in this economy, linked through operative bequests to the never-ending chain of his first-born descendants, and is thus part of an infinitely-lived family. Children who are not first-born, however, do not belong to any pre-existing dynasty, since they were not loved by their parents: they initiate the dynasty to which they belong. The rate at which new dynasties enter the economy is a reflection, in such an environment, of the proportion of children who are not loved, or not loved enough, by their parents*" (Weil, 1989).

proposition.” Furthermore, Weil (1989) constructed an example for finite horizons, for which Ricardian debt neutrality is not necessarily violated.

Besides the operative intergenerational transfers, the lifetime horizon and the way taxes and tax bases are distributed and perceived among generations, an important contribution adds the degrees of knowledge spillover across generations, as well as the substitutability between consumption and leisure, to the factors impacting equilibrium interest rates and Ricardian equivalence validation. In this way, Ni (1999) extended Diamond's OLG model, where capital is the only variable input, by allowing for capital and labour as inputs, and assuming a learning-by-doing knowledge-based growth economy in the spirit of Arrow (1962) and Lucas (1988). Diamond’s (1965) model, and others like it, imply that a government deficit always reduces savings and raises interest rates. However, Ni’s (1999) main model results show that the effect of government deficit on interest rates depends on the spillover of knowledge and the elasticity of labour supply. Precisely, in a neoclassical growth model with elastic labour supply and intergenerational spillover knowledge, public deficits may not necessarily raise real interest rates.[20]

The explanation for this is as follows: while a debt-for-tax swap reduces only the capital stock in the Diamond (1965) model, it reduces both the supply and demand of capital in Ni’s (1999) model. Then, with consumption and leisure being good substitutes, shifting the tax burden to the future reduces current savings, capital stock and the labour supply of the near future. The overall impact on real interest rates is a result of two opposite effects of reduction in the labour supply and the accumulation of knowledge[21] from one side and capital supply from the opposite side. The reduction in future labour supply and knowledge lowers the equilibrium real interest rate, while the reduction in capital increases it (as in Diamond, 1965). The first effect may partially offset and possibly overcome the second one.

To sum up, this section was devoted to the public debt effects in the neoclassical growth models of ILA and OLG. The most debated question in these models is the links between the validity of the Ricardian argument and the lifetime horizon of individuals. Indeed, if the presence of agents with an infinite lifespan in the ILA models (based on Ramsey, 1928) validates the Ricardian equivalence theorem, the OLG (deriving their core framework from Samuelson’s (1958) model and Diamond’s (1965) model) assign the validity of Ricardian equivalence to the links that tie the people who die and those who will be born (Barro, 1974; Blanchard, 1985; Buiter, 1988; Weil, 1989). Beyond debt neutrality and its prerequisites, the neoclassical models show that public debt raises interest rates, in turn crowding out capital and reducing welfare and utility. However, despite important contributions in refining the analysis of government debt effects related to the bequest motives and intergenerational altruistic transfers, the effects are difficult to assess empirically

[20] Furthermore, Ni (1999) suggests, using an empirical investigation, that the dynamics of the interest rate effect are difficult to assess, and the results of empirical studies of the interest rate effect of government deficits should be carefully interpreted.
[21] Government deficits reduce the current labour supply and slow down the accumulation of knowledge.

under the ILA and OLG models. This is a result, in particular, of the difficulty gauging the degree of such bequest motives between generations.

2.2. Government debt in endogenous growth models

Models of endogenous growth theory were developed to endogenize the role of externalities and their contribution to explaining the persistence of the long-term per-capita growth rate, as an alternative to the rival neoclassical Solow–Swan model (Solow, 1956; Swan, 1956).[22] The latter considers the role of such externalities, or what is assumed to be technical progress, as exogenous. Indeed, the steady-state growth rate in the Solow–Swan model is determined entirely by exogenous elements, and macroeconomic aggregates (capital, output and consumption) grow at a constant exogenous rate of the population growth, which makes the per capita corresponding quantities constant, and hence they do not grow. Therefore, according to Barro and Sala-i-Martin (2003), the main substantive conclusions about the long term are that steady-state growth rates are independent of the saving rate or the level of technology. Specifically, a model without technological change (like the Solow–Swan model) predicts that economies will converge to a steady state with zero per-capita growth as a consequence of the diminishing returns to capital. Solow's model also appeared to be “obsolete”, since the total factor productivity (TFP) measure estimated the share of growth explained by technical progress to be more than 50%, as reported by Jones and Romer (2010), or ranging between 50% and 70%, according to Hsieh and Klenow (2010). This constitutes an “empirical” argument for the emergence of endogenous growth models (Hulten, 2001; Aiyar and Feyrer, 2002; Fuentes and Morales, 2011).

With its standard framework, the Solow–Swan model was unable to explain the persistent per-capita non-zero growth rates in many developed economies, and hence was highlighted for missing the determinants of long-term growth. Thus, the crucial goal of the pioneers of the endogenous growth theory is to encompass other determinants of long-term growth. This includes broadening the concept of capital, in which the assumption of diminishing return to scale is avoided, to include other determinants as inputs in the process of production, such as human capital (Lucas, 1988; Romer, 1990), innovation, ideas and knowledge[23] (Grossman and Helpman, 1991; Jones, 1995, 2003), public goods and service flows (Barro, 1990), public capital and productive public capital (Futagami et al., 1993), public debt (Greiner, 2007).

To provide explicit contributions involving government spending and public debt in endogenous growth models, we select a benchmark of important contributions. Thus, Barro (1990) was the first

[22] In 1956 Solow and Swan published two distinct papers on the same issue, and their model is referred to as the Solow–Swan model, or often just the Solow model in reference to the more famous of the two economists.
[23] Technological progress is also viewed as a form of generating new ideas by which an economy could escape from diminishing returns to scale in the long term. Consequently, dealing with technological progress as endogenous within economic growth models, instead of exogenous, is an important strand of the endogenous growth theory. However, a technical discussion emerged on how to include ideas and some public goods in the neoclassical production function because of their non-rivalry characteristics (as for the case of ideas) and non-excludable properties (as for some public services: national defence, justice, law and order).

to introduce government services as flows in the AK modelling framework. This article triggered a series of theoretical extensions (Barro and Sala-i-Martin, 1992, 1997; Futagami et al., 1993; Turnovsky, 1997), as well as a variety of empirical contributions (Greiner, 2007; 2011; 2012; 2015; 2016; Ghosh and Mourmouras, 2004a, 2004b; Futagami et al., 2008; Maebayashi et al., 2017; Minea and Villieu, 2013; Yakita, 2008. The aim of the contributions was to establish the effect of public spending and government debt policies on productivity and long-term growth from a perspective of endogenous growth. Public spending encompasses a variety of expenditure and subsidies covering diverse sectors of health, education, research activities, research and development (R&D), public roads and infrastructure, defence and security, justice and law enforcement, and so on. Besides the direct intra-sector effects, some of this spending may also have externalities on other sectors producing knowledge, ideas and powering human capital by affecting their productivity. According to Corsetti and Roubini (1996), in addition to positive effects either on labour productivity or as rents generated proportionally by a fixed factor, as assumed in previous works (before their paper), productive public spending may also exert an external effect on the productivity of physical capital.[24] The assessment of the external effects of productive public spending is rather an empirical issue.

The Barro (1990) model highlights an explicit link between government policy and long-term economic growth in an endogenous growth model by incorporating government investment expenditure into the neoclassical production function with constant returns. The model studies a closed economy with infinite lifetime agents and inter-temporal preferences modelled by a utility function. The author justifies including government services as a separate input of the production function by the fact that private input is not a close substitute for public input. It is difficult to ensure some public activities through private firms as their charges are difficult to implement, as in the case of non-excludable services (national defence and the maintenance of law and order), or because the service is non-rival (ideas) or because external effects cause private production to be too low (as argued for basic education).

The model includes public consumption as an input in the production function, such as:

$$y = k\varphi\left(\frac{g}{k}\right) \tag{1}$$

where y, k and g are, respectively, the output per capita, the capital per capita and the government consumption of goods and services per capita. φ satisfies the conditions of positive and diminishing marginal products ($\varphi' > 0$ and $\varphi" < 0$). Considering φ, a Cobb-Douglas production function, yields:

[24] Internet services is an example of how the same public good might affect the productivity of either final goods and/or human capital accumulation (Corsetti and Roubini, 1996).

$$\frac{y}{k} = A\left(\frac{g}{k}\right)^{\alpha} \tag{2}$$

where A is a constant net marginal product of capital and $0 < \alpha < 1$. Furthermore, the model assumes a balanced government budget (tax-financed public services) by a flat-rate income tax such as:

$$g = T = \tau y = \tau A\left(\frac{g}{k}\right)^{\alpha} \tag{3}$$

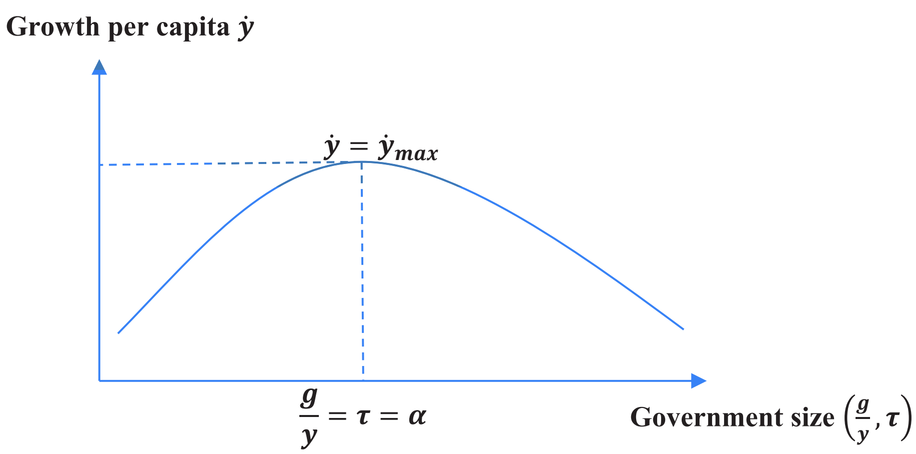
with T being the per capita amount of taxes.

Important implications under the previous assumptions result in different values of government size (g/y or τ) having different effects on the long-term growth rate. An increase in τ reduces the long-term growth rate, while an increase in g/y raises it through an increase in marginal productivity of capital ($\partial y/\partial k$). The two effects cancel each other out for the optimal government size equalizing government expenditure to output:

$$\frac{g}{y} = \tau = \alpha \tag{4}$$

Equation (4) corresponds to the maximum long-term growth rate.[25] The growth rate function of government size is an inverted U relationship (Figure 1). Hence, for a small government, the effect of raising expenditure g/y dominates the effect of raising tax rate τ, while for a large government size the negative effect of taxes on growth dominates.

Figure 1. The relationship between per-capita long-term economic growth and government size



[25] For a non-Cobb-Douglas production function, the maximum growth rate depends on the elasticity of substitution between per-capita government services and per-capita private capital. Similar conditions and results also apply for the saving rate.

As for the effects on utility, its maximization corresponds to the same conditions that maximize the economic growth rate if the elasticity of substitution of y with respect to g equals unity. Particularly in the case of the Cobb-Douglas production function and the same previous notations, government size that maximizes utility corresponds to the condition in equation (4). If the production function is not a Cobb-Douglas form, the relative size that maximizes utility exceeds the one that maximizes growth rate if, and only if, the magnitude of substitution between g and k is superior to unity.

If Barro (1990) considers the flow of public services in the production function, Futagami et al. (1993) build on this by considering the accumulated productive public capital as an input in the production function,[26] which generates a sustained per-capita growth rate in the long term. Futagami et al. (1993) argue that public investment stimulates aggregate production indirectly via the accumulated stock instead of flows, as in Barro (1990). Furthermore, the introduction of the productive public stock allows dynamic transitional effects analysis instead of being restricted to steady-state analysis, as in Barro (1990) and others. As a result, they show that a tax rate that maximizes the economic growth rate turns out higher than the one that maximizes utility. However, despite these enhancing elements attributed to Futagami et al.'s (1993) model, the latter still considers that the government budget is balanced at any time, as assumed in Barro (1990).

Therefore, instead of assuming that government is restricted to running a balanced budget in every period, as in Barro (1990), Futagami et al. (1993), Barro and Sala-i-Martin (1992; 1997) and many other previous works, subsequent research has omitted this assumption and tried to study fiscal policies under an unbalanced government budget.[27] In this area, Corsetti and Roubini (1996) consider productive public spending to assess optimal fiscal policy (public spending, tax and financial policies) in the same framework as endogenous growth models. They relax the balanced budget assumption to unbalanced budget constraint, thus allowing for government to borrow and lend. Furthermore, they incorporate in their model a separate human capital accumulation as a second sector contrary to many previous papers restricting investigation to one-sector models where public spending can only affect the productivity of the final goods sector. This allows for studying of the properties of government policies on both the final human and non-human capital sectors. In particular, they distinguish optimal tax rates for both types of capital under different

[26] This was signalled first by Arrow and Kurz (1970), but in a non-endogenous growth framework. Assuming diminishing returns to scale in private and public capital given an amount of labour services, the per-capita growth rate depends on the exogenous rate of technological progress alone.

[27] Considering an endogenous growth model where the history of debt affects the primary surplus of the government. Greiner (2014) shows that an economy with a balanced government budget is characterized by a unique balanced growth path. Inversely, with a permanent public deficit, the balanced growth path is either non-existent or non-unique and could be either stable or unstable. Moreover, Greiner (2015) shows that a balanced government budget yields higher balanced growth and welfare and lower inflation than a situation with permanent deficits, especially (for welfare) when the government does not put a high weight on stabilizing debt. In the absence of the latter condition, welfare effects hinge on the initial conditions of public debt.

assumptions on technology and distribution, and they analyse the welfare properties of public debt and assets.

The government service flows are introduced in the production sectors, and the effects of fiscal policies and debt are studied by deriving four models depending on whether: public spending is included as input in the output sector, affecting only the productivity of physical capital (Model 1); or affecting only the productivity of human capital (Model 2); or public spending as input in the human capital sector, affecting, respectively, the same variables (Models 3 and 4).

For some useful details, especially when government service flows enter as input only in the final production goods, Corsetti and Roubini (1996) consider an aggregate social production function in the style of Cobb-Douglas, assuming constant returns to scale to its three inputs, namely, physical capital, human capital and government flows of services, as follows:

$$Y_t = A(v_t K_t)^{\alpha\epsilon}(u_t H_t)^{1-\alpha}(G_t)^{\alpha(1-\epsilon)} \tag{5}$$

where v and u are, respectively, the fraction of total physical and human capital devoted to the production of final goods, and the productivity of public spending is decreasing in the parameter $0 < \varepsilon < 1$. It follows that the optimal government size is deduced by:

$$\frac{g}{y} = \alpha(1 - \varepsilon) \tag{6}$$

As $\varepsilon > 0$, the optimal government size in Corsetti and Roubini (1996) is less than the one provided by Barro (1990) in Equation (4): $\frac{g}{y} = \tau = \alpha$ superior to $\alpha(1 - \varepsilon)$. According to Corsetti and Roubini (1996), the optimal government size should be properly regarded as a result of the optimal choice of spending that holds with and without distortionary taxation, while Barro (1990) and others consider the (second-best) optimal choice to be the tax rate under an instantaneously balanced budget assumption.

Despite Corsetti and Roubini (1996) assuming an unbalanced government budget, they include public spending flows as productive input, as in Barro (1990), instead of accumulated public capital, as in Futagami et al. (1993).

Recently, many works[28] have concentrated on the issue of public debt, accumulated capital stock of public spending and, generally, fiscal policy and its effects on long-term growth and welfare, in an endogenous growth framework, particularly in the presence of unbalanced budget constraint with debt dynamics. For example, Ghosh and Mourmouras (2004a) extend Futagami et al. (1993) to the case of welfare-maximizing fiscal rules, in the presence of government debt. Thus, the public

[28] A non-exhaustive list includes among these works Greiner and Semmler (1999, 2000), Ghosh and Mourmouras (2004a, 2004b), Bräuninger (2005), Greiner (2007, 2012, 2015, 2016), Futagami et al. (2008), Yakita (2008), Minea and Villieu (2013) and Maebayashi et al. (2017).

to private capital ratio is lower under a golden rule of public finance (than under other fiscal regimes), minimizing crowding-out effects. However, steady-state welfare may be lowered in a less strict budgetary rule if public consumption rises, leading to crowding-out effects. In the same context, Futagami et al. (2008) construct an endogenous growth model with productive government spending where the government finances expenditure through income tax and government debt and puts a target level of government debt relative to the size of the economy. The model distinguishes two steady states: a high-growth steady state in which an increase in government bonds reduces the growth rate; and a low-growth steady state where an increase in government bonds raises the growth rate. These results are inverted in the case of an income tax increase. For the level of welfare, it is lower in the low-growth steady state than in the high-growth steady state.

Nevertheless, Minea and Villieu (2013) assign the existence of the two steady-state growth rates to the assumption of the public debt target as a ratio to private capital. Therefore, once the target has been defined in terms of public debt-to-GDP ratio, the model leads to a unique and determined balanced growth path. Maebayashi et al. (2017) build on the models of Futagami et al. (2008) and Minea and Villieu (2013) by considering the stock of capital investment rather than flows, as in those models. Hence, they derive an optimal target debt ratio that depends on the tax rates on wage income and consumption, as well as the public investment share in total government spending. The target debt ratio set by the Stability and Growth Pact (SGP) and Maastricht Treaty, namely 60%, is judged to be higher than the optimal level. Moreover, debt reduction based on expenditure cuts alone improves welfare. In particular, fiscal consolidation based on a target level of debt-to-GDP ratio rule (i.e. the well-known 60% rule) improves welfare, and the faster the pace of debt reduction, the greater this improvement is. Furthermore, fiscal consolidation based on expenditure cuts, jointly with a tax increase, does not always improve welfare. In this case, the welfare gains (if any) are lower than those under expenditure cuts only.

In the same context of targeted ratio of public debt, other authors study the sustainability of public finance. Thus, Bräuninger (2005) uses an endogenous growth model in the form of AK production function to determine a threshold public deficit ratio. Yakita (2008) builds on Futagami et al.'s (1993) production function to determine a sustainable threshold of public finance that increases in public capital stock. Consequently, a larger public capital helps to sustain public finance. Moreover, keeping the debt finance ratio invariable, the threshold of the debt-to-public-capital ratio increases with reduced public investment in GDP ratio. Increasing public capital ratios in Bräuninger (2005) and Yakita (2008) requires higher taxes and additional bond issuance, leading to higher interest rates and crowding-out effects.

Likewise, Greiner and Semmler (1999, 2000) relax the assumption of government balanced budget and allow for capital market borrowing by the government. Thereby, they analyse the effect of a deficit-financed increase in productive government spending following some predefined budgetary regimes. Hence, fiscal policy effects are significantly determined by such budgetary rules. Likewise, Greiner (2007) analyses an endogenous growth model with public capital and sustainable public debt.[29] The model is used to derive the necessary conditions for the existence of a sustainable balanced growth path and to analyse the growth effects of deficit-financed increases in public investment in the sustainable balanced growth path, as well as along the transition path. Additionally, in a model with elastic labour supply and a government sector in which government levies a distortionary income tax and issues bonds to finance lump-sum transfers and non-distortionary public spending, Greiner (2012) shows that the higher the debt ratio, the smaller the long-term growth rate whenever public spending is adjusted to fulfil the government inter-temporal budget constraint. However, the public debt ratio has no effect on the balanced growth rate if the adjustment is on lump-sum transfers.

Analysing the effects of public debt in an endogenous growth model with productive[30] public spending, Greiner (2015) shows that higher debt accompanies smaller long-term growth. Moreover, discretionary policy, in general, violates the inter-temporal government budget constraint along a balanced growth path. A balanced government budget gives a unique saddle-point stable growth path, while a rule-based policy can lead to two saddle-points stable balanced growth, depending on the inter-temporal elasticity of substitution of consumption and on the primary surplus policy. For Greiner (2016), an endogenous growth model with public educational spending shows that the balanced budget policy[31] and the policy with a slight deficit yield higher growth than a debt policy where public debt and GDP grow at the same rate. Furthermore, for high initial debt ratios and low inter-temporal elasticity of substitution, a strong deficit policy yields lower welfare than a balanced budget and a slight deficit policy.

In summary, if the neoclassical ILA and OLG models have been interested in the long-term effects of government fiscal policies (debt and taxation) on the saving–spending behaviour of individuals and generations, through operative linkages and transfers, the endogenous growth models have endogenized such policies to assess their effects on the steady-state growth path. Furthermore, including debt dynamics and/or public capital stock, endogenous models eventually allow for tracing of the transitional dynamics effects of fiscal and debt policies that could not be ensured by the ILA and OLG models. The extensions and development of such models triggered a prolific discussion about the composition of public expenditure (productive as input versus non-productive as utility), the non-rivalry and non-excludable goods, and the associated effects on physical and human sectors for each type of public goods. Accordingly, the differentiated effects between the types of expenditure induce different policies for the government. Despite these important

[29] Public debt sustainability is assured by assuming the ratio of the primary surplus to gross domestic income to be a positive linear function of the debt to income ratio.
[30] "*The productive public spending can be thought of as encompassing very different types of publicly provided goods and services, such as justice, enforcement of law and contracts, police services, educational services and government research activities*" (Corsetti and Roubini, 1996).
[31] Greiner (2011) compares the outcome of three budgetary rules: the balanced budget rule, a budgetary rule where debt grows in the long term but at a rate lower than the balanced growth rate, and a rule where public debt grows at the same rate as all other economic variables and where the inter-temporal budget constraint is fulfilled.

contributions, further enhancement remains, especially in considering the heterogeneity of agents' behaviour and welfare, which was also an important drawback of the ILA and OLG models (Mankiw, 2000; Maebayashi et al., 2017). Explicitly, a population could embrace a part of the agents with Ricardian behaviour, while the other part behaves following a rule of thumb.

2.3. Government debt in the new Keynesian models and the positive approach of public debt

This section presents a summary of some very recent new Keynesian models involving public debt and fiscal policy, as well as the public debt effects in the models, considering a positive approach where the political regime impacts the trajectory of public debt.

2.3.1. Government debt in the new Keynesian models

Instead of assuming that all taxes are a lump sum, especially in the assumptions about overlapping generations models, recent works have considered optimal fiscal policy, assuming distortionary taxes in the class of new Keynesian models in which social welfare is implied from a consumer utility function. The pioneering works in the new Keynesian modelling framework are principally those of Christiano et al. (2005) and Smets and Wouters (2007). Government debt effects, especially in the neoclassical models, are studied in the long term and steady-state equilibrium, especially in the ILA and OLG models and some endogenous growth models lacking transitional dynamics. However, the recent generation of the new standard Keynesian models or dynamic stochastic general equilibrium (DSGE) models have studied optimal government policies (monetary and fiscal policies) and fiscal consolidation issues considering economic fluctuations and shocks (in the short and medium term), in which public debt is set in many of these models to zero.

Nevertheless, some authors have studied the trajectory of public debt in relation to committed government actions. Particularly, optimal public debt would follow a random walk process whenever the government can achieve a time-inconsistent policy commitment (Benigno and Woodford, 2003; Schmitt-Grohe and Uribe, 2004). Specifically, the latter study the implications of price stickiness for the optimal degree of price volatility. The model considers a government issuing non-distortionary taxation and can only issue nominally risk-free debt. Specifically, under the assumption of price stickiness in this class of models, the government (social planner) chooses to rely more heavily on changes in income tax rates rather than using surprises as a shock absorber of unexpected innovations in the fiscal budget. The distortions introduced by tax changes are diminished by spreading them over time, which induces a near random walk property in tax rates and public debt.

In the context of new Keynesian models augmented by the government's budget constraint, where public spending is financed by distortionary taxes and/or debt, Leith and Wren-Lewis (2013) analyse the optimal response of government debt to shocks, focusing on the type of involved time-inconsistency policy and its implications for discretionary policies. Like the previous research, they find that the optimal pre-commitment policy allows debt to follow a random walk path in the steady state. However, they show that, under a sticky prices framework, governments are tempted to use their monetary and fiscal policy instruments to change the steady-state level of debt in the initial period. The debt will be curved to its initial efficient steady state to encounter this temptation and therefore deter public debt from following the random walk path if following shocks. The new steady-state debt equates the original (efficient) debt level even though there is no explicit debt target in the government's objective function. Analytically, they show that the debt stabilization instrument crucially depends on the degree of nominal inertia. Furthermore, the size of the debt stock, and the welfare consequences of introducing debt, are negligible for pre-commitment policies but can be significant for discretionary policies.

Furthermore, a few studies examine the effects of high debt in fiscal consolidation actions related to the impact on the magnitude and/or sign of the fiscal multipliers. For example, Mayer et al. (2013) use a new Keynesian model to analyse whether, and how, the presence of positive levels of government debt in the steady state influences the responses of macroeconomic variables to a government spending shock. The model assumes that a fraction of the household sector is characterized by rule-of-thumb behaviour, as in Galí et al. (2007). They show that large levels of government debt in the steady state significantly influence the sign and size of short- and medium-term fiscal multipliers, which therefore depend substantially on the horizon at which the multiplier is evaluated. Furthermore, there is an interaction between the dynamics of the inflation rate and the debt level in real terms, which is absent in standard new Keynesian models in which government debt is assumed to be equal to zero in the steady state. Overall, in the presence of permanent government debt, the effect of fiscal policy on macroeconomic variables becomes difficult to predict over time.

2.3.2. The positive approach of public debt

In the previous section the ILA and OLG models were generally developed by economists in an environment where governments, being benevolent social planners, maximize the utility of their population. These models are classified under what is referred to in the economic literature as "the normative approach" (Alesina and Tabellini, 1990), the "tax smoothing" theory of the government budget (Alesina and Perotti, 1994) or "the equilibrium approach to fiscal policy" (Roubini and Sachs, 1989). The normative theory of debt and fiscal policy considers public debt as a means of smoothing consumption by distributing tax distortions over time (Barro, 1979; Turnovsky and Brock, 1980; Lucas and Stokey, 1983). The core models of this theory assume, in general, a closed economy without capital where the government is a "benevolent social planner" that maximizes the utility of a representative agent who consumes, works and saves with the same infinite (simplified) time horizon of both government and representative agent (Alesina and Perotti, 1994).

However, for advocates of the positive approach (Roubini and Sachs, 1989; Alesina and Tabellini, 1990; Grilli et al., 1991), normative theory, despite explaining the behaviour of debt in many advanced OECD countries, has been challenged by the rapid accumulation over time of debt in almost all developed countries. Therefore, it is unable to provide a complete explanation for such a phenomenon or explain the differences in policies pursued by different countries with comparable economic conditions.

Alternatively, a positive approach has been the subject of modelling government fiscal policy, particularly debt policy, in the field of the new political economy of public debt.[32] This approach is particularly interested in the implied impacts of political process on shaping the path of government debt. The positive approach considers public debt as a state variable used by each government as a strategy to influence its successor's choices or to shape private economic agents' expectations (Persson and Svensson, 1989; Alesina and Tabellini, 1990).

In this way, Roubini and Sachs (1989) notice that, in several industrialized countries, issues of political management in coalition governments fall behind the slow rate at which fiscal deficits were reduced during the 1975–85 period. In particular, during this period weaker governments[33] had a clear preference for larger deficits. Similarly, Grilli et al. (1991) focus on the role played by public institutions in offering constraints and incentives that determine the actions of governments. Governments' ability to handle growing deficits and debt issues is influenced by the electoral practice and political process. Grilli et al. (1991), following Roubini and Sachs (1989), note that, in countries with an electoral system favouring many small political parties, governments generally have short horizons and therefore act myopically to avoid tackling the hard choices.

Persson and Svensson (1989) consider the level of public debt as the state variable that gives the current government an instrument to control a rival future government. They compare, in a two-period perfect-foresight framework, the policy of a conservative government (one that prefers less debt and deficit), which is certain to be succeeded by a liberal government (a more expansionary government), with the policy where it is certain that it will remain in power. As a result, a conservative government may borrow more if it knows it will be succeeded by a liberal government than it would once certain of remaining in power in the future. Obviously, a conservative government will collect less tax and leave more public debt than the successor would prefer. This increases the conservative government's consumption more than if it remained in government, while the liberal government (successor) with high debt and constrained resources reduces consumption more than it would if it ran alone. Thus, the time-consistent level of

[32] Alesina and Perotti (1994) provide a survey of the political economy models of budget deficit organized into six groups: 1- models based upon opportunistic policy-makers and naïve voters with "fiscal illusion"; 2- models of intergenerational redistribution; 3- models of debt as a strategic variable, linking the current government with the next one; 4- models of distributional conflicts within social groups and/or political parties in coalition governments; 5- models of geographically dispersed interests; and 6- models emphasizing the effects of budgetary institutions. In our case, we are especially interested in the third group, particularly the contributions of Persson and Svensson (1989) and Alesina and Tabellini (1990).

[33] Weaker government is characterized by a short average tenure and by the presence of many political parties in the ruling coalition.

government consumption is somewhere between the two outcomes that the two governments would prefer if ruling on their own.

Meanwhile, the Persson and Svensson (1989) approach, known as a principal-agent problem, with the conservative government being the principal and the liberal successor government being the agent, simplifies the reality by assuming that the ruling government knows with certainty that it will be succeeded by a more liberal government. They also assume the homogeneity of governments' preferences towards all public goods but different preferences for different levels of the same public good. In this regard, Alesina and Tabellini (1990) constitute an advancement in introducing uncertainty about the nature and spending behaviour of successive governments. They also consider different preferences for different items of public expenditure, while Persson and Svensson (1989) focus on different levels of the same public good.

Therefore, to properly understand the debt build-up and deficits in several industrialized economies, Alesina and Tabellini (1990) adopt a positive theory by removing the assumption that fiscal policy is set by a benevolent social planner who maximizes the welfare of a representative consumer. Specifically, their findings suggest that differences in political institutions, leading to different debt policies in different countries, or in the same country at different points in time, help to explain the debt trajectories over time and across countries. Their model[34] is derived from Lucas and Stokey (1983). In particular, Alesina and Tabellini (1990) show that debt accumulation and deficit are accentuated by the alternation of elected governments.

Explicitly, they compare the outcome of debt accumulation and deficit in a situation where governments alternate versus an outcome resulting from a social planning[35] government supposedly elected forever. Specifically, a disagreement between different governments on the composition of spending between public goods results in a deficit bias and hence an accumulation of debt higher than would be the case in the situation of a social planner. As explained by Alesina and Tabellini (1990): "*The level of debt left to the last period is larger in a democracy than with a social planner; namely the social planner would choose to balance the budget in both periods, while either one of the two parties choose to run a budget deficit in the first period leaving a positive amount of debt to be repaid in the last period. In this sense, the electoral uncertainty creates a sub-optimal deficit bias. This bias is stronger for the party with the smaller probability of reappointment.*"

[34] The model assumes mainly a constant population of identical individuals with the same time horizon, acting as consumers, workers and voters. Individuals differ only by their preferences for public goods, supplied by the government and financed by means of distortionary taxes on labour. The government is elected democratically and is chosen among two political parties, each maximizing the utility function of its electorate. Disagreements between the governments are viewed as differences about social welfare.

[35] "*A social planner: 1) do[es] not face elections; thus, she is reappointed with probability 1 in each period, 2) her preferences are a weighted average of the preferences of the citizens*" (Alesina and Tabellini, 1990).

Furthermore, the equilibrium level of government debt is higher with: 1/ a higher degree of polarization between alternating governments; and also 2/ with more likelihood of the current government not being re-elected. Moreover, as the ruling government is unable to curve the taxation and expenditure policies of its successors (whether the successor belongs to the same or the opposing party), the law of motion of public debt is the only way in which the fiscal policy of the ruling government can impact the policies of its successors.

Another important result is related to the probability of re-election. In such an uncertain environment, both governments have the same incentive for increasing debt, not certain of being re-elected, and they restrict the next period's public consumption by increasing borrowing for the current ruling period. This leads to a bias towards a larger deficit and higher debt for both governments, which helps to explain the accumulation of public debt in advanced democracies.

2.4. Discussion

The first section reviewing the effects of public debt through the ILA and OLG models leads to two principal results. The infinitely lived dynasties models, derived from Ramsey (1928), and adopted by Barro (1974) and others, validate the Ricardian equivalence proposition for which public debt is neutral. It is worth mentioning that the Ricardian equivalence proposition works assuming economic agents' rational expectations. Accordingly, the previous models show that economic agents (some, but not necessarily all) react to fiscal policy redistribution of the tax burden among generations through their bequest motives inducing operative transfers to smooth the pattern of consumption over time. However, because the OLG models of Diamond (1965) and others lack such bequest motives, a government debt issuance affects the wealth of generations by raising real interest rates, hence crowding out capital and reducing the steady-state utility.

Despite these important contributions for economic theory in assessing government debt and fiscal policy effects, these two types of model have been subject to criticism. Mankiw (2000) criticized the two modelling approaches, arguing against their adequacy and satisfactory role for analysing fiscal policy. Accordingly, the author is, first, sceptical regarding the assumption that "households smooth their consumption over time" that is adopted by both versions of the model. This assumption is far from perfect, according to Mankiw (2000). In particular, current income significantly impacts consumer spending, as many consumers are far from following completely rational expectations, instead adopting *rule-of-thumb*[36] behaviour (non-Ricardian behaviour) in their spending. Second, some individuals may enjoy long lifetime horizons (due to bequest motives), while others with short time horizons fail to smooth their consumption and accumulate wealth. Third, many households have net wealth near zero (a striking fact reported in the data),

[36] For example, Galí et al. (2007) use a new Keynesian model to empirically test government spending effects on consumption, involving *rule-of-thumb* agents that only have access to contemporaneous labour income for consumption, and *Ricardian agents* that can smooth consumption by accumulating capital.

and hence no savings, which makes them unable to follow inter-temporal consumption smoothing, as reported by the Barro-Ramsey or Diamond-Samuelson models.

Thus, for Mankiw (2000), a better model would allow for such heterogeneous behaviour that is apparent in the data. In this regard, Mankiw (2000) formulated an alternative theory mentioned as "savers–spenders theory of fiscal policy" to address the neoclassical shortcomings in public finance policies. This theory shows, in particular, that, even though government debt does not affect steady-state capital stock and income, it disrupts income distribution and consumption in the steady-state path and, in turn, raises inequality between spenders and savers. Specifically, a higher level of debt yields higher taxation to compensate for the interest payments on the debt. However, the taxes are on both savers and spenders, while the interest payments on debt fall on the savers' side. Therefore, a higher level of debt increases the steady-state income and consumption for savers (with already higher initial income) and lowers it for spenders (with initially a lower income), which raises inequality between the two groups.

The "savers–spenders" theory of Mankiw has influenced many empirical researchers on fiscal policy trying to consider the behaviour following the rule of thumb, especially in the new Keynesian models. However, this type of modelling has also not been immune to criticism. In this way, Chari et al. (2009) show that this class of model is not yet useful for public policy analysis. Chari et al. (2009) base their critiques on the model of Smets and Wouters (2007), which constitutes a fundamental reference for many recent contributions and policy-makers using the new Keynesian analysis framework. Accordingly, the Smets and Wouters (2007) state-of-the-art model adds many free parameters to these models, yielding to shocks that are dubiously structural, as well as many features that are not consistent with the microeconomic evidence.

The drawbacks of the new Keynesian modelling framework were recognized especially after the 2008 financial crisis and are frequently cited in several publications. In this way conferences with different slogans (rethinking macroeconomics, rebuilding macroeconomic theory, etc.) have gathered economists from around the world in an attempt to discover why these models failed to warn about the disaster of the financial crisis. In this way an important project (the Rebuilding Macroeconomic Theory Project) asked a number of leading economists to describe how the benchmark new Keynesian model might be designed after the financial crisis. Fifteen important articles and contributions (Vines and Wills, 2018a, 2018b; Blanchard, 2018; Wren-Lewis, 2018; Stiglitz, 2018; Wright, 2018; Reis, 2018; Krugman, 2018; Carlin and Soskice, 2018; Ghironi, 2018; Haldane and Turrel, 2018; Lindé, 2018; Hendry and Muellbauer, 2018; Mckibbin and Stoeckel, 2018) on this question were published in the *Oxford Review of Economic Policy* (2018, vol. 34 (1–2)). The authors disagreed that the new Keynesian models benchmark of Smets and Wouters (2007) should not constitute the starting point for the newly designed model. Nevertheless, they agree that the core model should consider four elements, as described in Vines and Wills (2018a). The core model should, in particular: i) incorporate financial frictions rather than assuming that financial intermediation is costless; ii) relax the requirement of rational expectations; iii) introduce heterogenous agents; and iv) underpin the model with more appropriate micro-foundations.

3. The theoretical framework

3.1. The choice of an endogenous growth model

We showed in Section 2 that the ILA models, derived from Ramsey (1928), and adopted by Barro (1974) and others, validate the Ricardian equivalence proposition for which public debt is neutral. The latter proposition holds in an environment of perfect information assuming economic agents' rational expectations. Therefore, such models show that economic agents (some, but not necessarily all) react to fiscal policy redistribution of the tax burden among generations through their bequest motives inducing operative transfers to smooth the pattern of consumption over time. However, because the OLG models of Diamond (1965) and others lack such bequest motives, a government debt issuance affects the wealth of generations by raising real interest rates, in turn crowding out capital and reducing steady-state utility.

Despite their important contributions to the economic theory assessing government debt and fiscal policy effects, the ILA and OLG models have been subject to criticism, to some extent diminishing their contribution. Mankiw (2000) criticized the two modelling approaches, arguing against their adequacy and satisfactory role in analysing fiscal policy for several reasons:

- ***First***, according to Mankiw (2000), the assumption that "households smooth their consumption over time" adopted by both versions of the model does not seem convincing. Specifically, many consumers are far from following completely rational expectations and instead adopt the *rule-of-thumb* behaviour (non-Ricardian behaviour) in their spending; hence, current income significantly impacts consumer spending. For example, Galí et al. (2007) use a new Keynesian model to empirically test the government spending effects on consumption, involving *rule-of-thumb* agents that only have access to contemporaneous labour income for consumption, and *Ricardian agents* that can smooth consumption by accumulating capital.
- ***Second***, some individuals may enjoy long lifetime horizons (due to bequest motives), while others with short time horizons fail to smooth their consumption and accumulate wealth.
- ***Third***, many households have net wealth near zero (a striking fact reported in the data), and hence no savings, which makes them unable to follow inter-temporal consumption-smoothing, as reported by the Barro-Ramsey or Diamond-Samuelson models.

For Mankiw (2000), a better model would allow for such heterogeneous behaviour as is apparent in the data. He therefore formulated an alternative theory mentioned as "savers–spenders theory of fiscal policy" to address the neoclassical shortcomings in the public finance policies. This theory shows that, even though government debt does not affect the steady-state capital stock and income, it disrupts income distribution and consumption in the steady-state path and, in turn, raises inequality between spenders and savers.

The “savers–spenders” theory of Mankiw has influenced many empirical researchers on fiscal policy trying to consider the behaviour of following the rule of thumb, especially in the new Keynesian models. However, the new Keynesian type of modelling has also not been immune to criticism. In this way, Chari et al. (2009) show that this class of model is not yet useful for public policy analysis. Chari et al. (2009) base their critiques on the model of Smets and Wouters (2007), which constitutes the fundamental reference for many recent contributions and policy-makers using the new Keynesian analysis framework. Accordingly, the Smets and Wouters (2007) state-of-the-art model adds many free parameters to these models, yielding to shocks that are dubiously structural, as well as many features that are not consistent with the microeconomic evidence.

Based on the previous critiques, the endogenous growth modelling framework seems to be a suitable candidate for our approach. The endogenous growth models via their productive sector (generally the Cobb-Douglas production function) are extremely flexible in encompassing many other factors to explain the per-capita long-term growth rate. Therefore, fiscal policy variables (productive government expenditure or accumulated government capital) are easily integrated to the production function to assess the effects of such variables in the steady-state (long-term) path and transitional dynamics.

3.2. Justification of government capital stock and government expenditure flows in the production function

Many studies have considered the issue of productive government spending, debt and fiscal policies, and their effects on long-term economic growth and social welfare in the context of endogenous growth models. We follow this line of research for several reasons and try to avoid many important issues. We mainly follow the approach of Barro (1990) and Barro and Sala-i-Martin (2003), but instead consider the public accumulated capital in the production sector rather than the flow of goods and services provided by the public sector. We also consider human capital, as in Corsetti and Roubini (1996), which could be an interesting measurable input. Our approach differs from Barro (1990) and Corsetti and Roubini (1996), as the latter use productive government flows in the production function, while our study uses accumulated public capital, as in Futagami et al. (1993) and Maebayashi et al. (2017).

In fact, we judge that models (either theoretical or empirical) using government expenditure flows instead of public capital stock lack some logic and compatibility with the framework of the Cobb-Douglas production function, especially when we consider the government as the production sector, which leads to considering productive government expenditure. This results in missing an important contribution of the earlier accumulated stock of public expenditure. As the model of production function, and others considered in the endogenous growth framework, are non-dynamic (no lag or inertia of the endogenous variable is present in the explanatory variable), considering the stock of capital ensures the dynamic effects of public expenditure. This means, economically, that the earlier flow contributions in the output are considered. However, considering only current flows in the production function implies that one considers the earlier accumulated flows (which builds the public stock) fully consumed or as if depreciated with a 100% depreciation rate.

Similar earlier critiques have been addressed to the Scully (1996, 1999, 2003) model calculating growth-maximizing of the government size. The Scully model formulae follow a form similar to the Cobb-Douglas framework, linking the current output to the first lagged government flows of expenditure and the first lag of output. Furthermore, assuming that the government budget is balanced and, with the government budget constraint equalizing government expenditure flows to the taxes represented as a share of the output, this leads to the following model:

$$Y_{it} = \beta_0(\tau Y_{it-1})^{\beta_1}[(1-\tau)Y_{it-1}]^{\beta_2} \tag{7}$$

with Y as output, τ as the lump sum tax rate, and t as the time period. This model is used to derive an optimal government size (equivalent to optimal taxation rate, as the balanced budget constrained is assumed) represented by the taxation rate as:

$$\tau = \frac{\beta_2}{\beta_1+\beta_2} \tag{8}$$

The Scully model form is highlighted as having many drawbacks and producing spurious regressions by many authors (Chapple, 1997; Easton, 1999; Kennedy, 2000 and Hill, 2008). Kennedy's (2000) critique is related to the unfounded relationship of this particular production function, which he says as if the public capital were totally used up every year. Moreover, Hill (2008) shows that this model should use the lagged tax rate in the previous equation instead of a current tax rate. When correcting this error, the growth-maximizing size of the state varies between 9% and 29% for the United States data, while Scully (1996) reports a government size of 19% for the same country.

Beyond that, despite many studies enriching the production function with other variables without paying attention to the nature of these variables, the introduction of stocks rather than flows is more reasonable and in conformity with the logic standard form of the neoclassical production function, which has microeconomic foundations.

Furthermore, this issue is emphasized further when we go through the details of the data of such flows. Indeed, a large part of aggregate government expenditure flows, introduced in the production function, are generally public employees' wages and salaries, direct transfers and subsidies to families and other public and private institutions. Except for some transfers to the latter (which are generally small compared to the total) that could help in the production process of these institutions, wages and salaries and transfers to families and price subsidies go directly to the households' income and constitute an input to their behavioural consumption function (or utility function). Thus, it is logically more suitable to consider it in relation to the latter instead of including it in the production function.[37]

An interesting issue that is difficult to measure, and which was mainly invoked by what is known as the positive approach of public debt in the domain of the political economy, is the crucial role of institutions in economic growth that uses debt and/or deficit as an instrument variable to curve future governments' (opposing) decisions and economic agents (Alesina and Tabellini, 1990). Considering institutions as a sort of game theory between succeeding governments, or between governments and their citizens in a normative modelling approach, is difficult. However, we could also take the role of institutions as endogenous rather than exogenous. One way is to augment the production function by a parameter of the government constraint representing the quality of the institutions. However, the role of institutions could also be considered by encompassing a formula or a parameter describing the inherent discretionary policies of the government, as in Maebayashi et al. (2017).

In the following section, we present the model.

3.3. The model

3.3.1. The productive sector

We use the production function to describe the relationship between accumulated public capital and real GDP. The production function can take different specifications, such as the constant elasticity of substitution (CES) or a trans-log production function, which under some specific restrictions can be reduced to the Cobb-Douglas production function. The latter is a special case of the former; in other words, the former are more flexible forms. The CES, for example, does not require the assumptions of perfect competition and profit maximization.[38]

We consider the Cobb-Douglas production function, which is well grounded in economic theory and mainly used in practice, to be easy to estimate, and it has good empirical properties (Razzak and Bentour, 2013). The equation[39] is as follows:

$$Y_t = A.{K_{pt}}^{\alpha} K_{gt}^{\beta} \tag{9}$$

[37] Government consumption posts and public investments are defined by the system of national accounts (SNA, 1993 and SNA, 2008) and the classification of expenditure by functions of the government, are categorized by the OECD classification (COFOG).
[38] "*Kmenta (1967) shows that estimating these flexible forms is not really difficult, except that they require [a] large number of observations because there have more parameters to estimate than in the Cobb-Douglas. Furthermore, an omitted variable problem may also be present. The omitted variable problem results in biased and inconsistent OLS parameter estimates*" (Razzak and Bentour, 2013).
[39] The time in all our equations is labelled by the subscripted lowercase letter "t" and could be omitted in case no confusion arises.

where Y_t, K_{pt} and K_{gt} are, respectively, the per capita of output, and the per capita of private and public capital. A is a constant designating technical progress, and α and β are shares in the national income of, respectively, private and public capital stock (also corresponding to elasticities). Private and public capital are evolving according to the following inventory equations:

$$K_{p,t} = (1 - \delta_{p,t-1})K_{p,t-1} + I_{p,t} \tag{10}$$

$$K_{g,t} = (1 - \delta_{g,t-1})K_{g,t-1} + G_t \tag{11}$$

where $I_{p,t}$ and G_t are, respectively, the private and public flows of investments and $\delta_{p,t-1}$ and $\delta_{g,t-1}$ are, respectively, the depreciation rate of private and public capital. To simplify, we assume the same constant depreciation rate for the private and public sectors ($\delta_{p,t-1} = \delta_{g,t-1} = \delta$)

Defining r_g as the productivity of public capital, the first-order condition derivation yields for public capital:

$$r_{g,t} = \frac{\partial Y}{\partial K_g} = \frac{\Delta Y}{\Delta K_g} = \beta \frac{Y}{K_g} \tag{12}$$

For a production function with constant returns to scale, $\beta = 1 - \alpha$, we have: $r_g = (1 - \alpha)\frac{Y}{K_g}$.

Equation (12) is equal to unity in the optimal steady state path ($r_g = 1$), which directly yields a constant optimal government size, as in Barro (1990) and Corsetti and Roubini (1996), when considering the flows of government services and not public capital. In our case, we prefer to study the economies considering that not all are in the steady state. This leads to a government expenditure size that depends on the return to public capital and its elasticity, which may differ between countries as a result of the differences in public governance and public expenditure productivities.

From equation (11) we have:

$$\frac{\Delta K_{g,t}}{\Delta Y_t} = \frac{-\delta K_{g,t-1}}{\Delta Y_t} + \frac{G_t}{\Delta Y_t} = \left[\frac{-\delta K_{g,t-1}}{Y_{t-1}}\right]\frac{Y_{t-1}}{\Delta Y_t} + \frac{G_t}{Y_t}\frac{Y_t}{\Delta Y_t} \tag{13}$$

From equation (12) we have the term $\frac{\Delta K_{g,t}}{\Delta Y_t} = \frac{1}{r_{g,t}}$ and $\left[\frac{-\delta K_{g,t-1}}{Y_{t-1}}\right] = \frac{-\delta\beta}{r_{g,t-1}}$. Putting $\gamma = \frac{\Delta Y_t}{Y_{t-1}}$ (which yields, $\frac{1+\gamma}{\gamma} = \frac{\Delta Y_t}{Y_t}$) as the nominal economic growth rate, and $\frac{G_t}{Y_t} = g_t = \tilde{g}_t$ the "potential"[40] government productive expenditure share to GDP, we obtain:

[40] We used the name "potential", as it is derived from a first-order condition.

$$\frac{1}{r_{g,t}} = \frac{-\delta\beta}{\gamma}\frac{1}{r_{g,t-1}} + \frac{1+\gamma}{\gamma}\tilde{g}_t \tag{14}$$

Or equivalently:

$$\tilde{g}_t = \left[\frac{\gamma}{1+\gamma}\right]\frac{1}{r_{g,t}} + \left[\frac{\delta\beta}{1+\gamma}\right]\frac{1}{r_{g,t-1}} \tag{15}$$

Neglecting the depreciation rate yields:

$$\tilde{g}_t = \left[\frac{\gamma}{1+\gamma}\right]\frac{1}{r_{g,t}} \tag{16}$$

Equation (16) implies that the potential productive government investment depends on the economic growth rate, as increasing the function of growth as the derivation according to the growth rate is positive, and inversely on public capital expenditure productivity. Hence, the higher the growth rate (higher potential of GDP also), the higher the potential public investment. Furthermore, the higher the productivity (the return $r_{g,t}$) of public capital, the lower the potential public investment.

However, once the depreciation rate has been accounted for, the effect of economic growth on the size of the government depends on the current and past productivity of public capital, as well as the elasticity and the depreciation rate parameters. The derivation of the productive share on the growth rate from equation (15) yields: $\frac{\partial g}{\partial \gamma} = \frac{1}{r_{g,t}}\frac{1}{(1+\gamma)^2}\left[1 - \frac{\delta\beta\, r_{g,t}}{r_{g,t-1}}\right]$, and the algebraic sign of this quantity depends on the sign of the term $\left[1 - \frac{\delta\beta\, r_{g,t}}{r_{g,t-1}}\right]$.

Furthermore, the optimal government expenditure is an endogenous parameter here and not a constant one, as raised by Barro (1990) or Corsetti and Roubini (1996) (named the size of the government for these authors). The assumption made by these authors is that the marginal product of public capital is equal to one in the optimum, leading to a "constant government size". This is also due to their consideration of government flows instead of public capital, which ensures this relationship.

Proposition: In an endogenous growth framework with the Cobb-Douglas production function encompassing public capital instead of government flows, the potential productive government investment size is an endogenous parameter of economic growth and public capital productivity.

3.3.2. The human capital sector

There is a thorough body of literature on technology diffusion where human capital is an essential element. The theoretical approaches are grounded in Nelson and Phelps (1966), Grossman and

Helpman (1991) and Barro and Sala-i-Martin (1997), while the empirical literature widely cites, as examples, the works of Benhabib and Spiegel (1994) and Borensztein et al. (1998). Human capital can either be an additional factor of production (Mankiw et al., 1992) or a factor influencing technical progress in the production function (Razzak and Bentour, 2013). Either way, it will appear as an additional repressor. Therefore, we consider having a measure of the stock of human capital as an additional regressor in the production function. The production function takes the form of Corsetti and Roubini (1996), except that here we consider the stock of public capital instead of flows of government services:

$$Y = A.{K_{pt}}^{\alpha\varepsilon}{H_t}^{1-\alpha}K_{gt}^{\alpha(1-\varepsilon)} \tag{17}$$

where $\mathrm{H_t}$ is human capital and the other variables are as previously defined.

3.3.3. The quality of public institutions

North (1990) defines institutions as "*the rules of the game in a society or, more formally, [they] are the humanly devised constraints that shape human interaction*". This characteristic of *devised human constraints* emphasizes the role of the endogenized character of institutions compared to external/exogenous factors outside human control (geography, for example). Their effects (*shape human interactions*) are mainly to shape the behaviour of humankind, thus directly impacting economic agents' incentives to invest and consumer choices. Following this effect on the incentives of economic agents, which affects the economic output of their actions, many authors have raised and emphasized the role of institutions in long-term economic growth and economic development (Barro, 1996; Acemoglu et al., 2001; Rodrik et al., 2004; Acemoglu and Robinson, 2010). Such a role is embedded (and hence endogenous) in different forms of dynamic interaction between economic, organizational, political and social factors (Aoki, 2007). The fiscal policy sector, in general, and the public debt/deficit management issues, in particular, are very important domains of public policy that are heavily determined and impacted by the institutional framework of the country.

The institutional framework plays its role in two dimensions. The first is related to the quantitative effects of government fiscal variables, which is summarized as the dimension size via the amount of expenditure and investments allocated by the government. The second is related to the qualitative aspects of government actions, which could be summarized by the quality of institutions encompassing the effects of many areas of government reform, programmes and actions (economic freedom, justice, rule of law, order and security, enforcing contracts, protecting investors, property rights, etc.). The dimension size, proxied by the stock or flows of government expenditure, has been embedded in models of endogenous theory, as described in Section 2.2 (Barro, 1990; Futagami et al., 1993, 2008; Corsetti and Roubini, 1996). However, the second aspect of the institutions relates to their quality in its broad sense and is not easy to observe, although this plays a crucial role in shaping the effects of the institutional framework on fiscal policy input (government expenditure, government debt, deficits, etc.) and also output variables (economic growth, society welfare, income distribution, etc.).

Modelling the quality of public institutions is a challenging and complex task for economists. So, while the economic models and the endogenous growth theory struggle to clearly include the institutions' role in the production process, the task is hardly emphasized, as the political side of the institutions plays an important role that is difficult to consider. The political character of the institutions implies removing the frontiers between the two disciplines: economic science and political science. Nevertheless, some economists have attempted to include empirically the role of institutions considering the data produced on governance indicators from international institutions, such as economic and political freedom of the heritage foundation, worldwide governance indicators of the World Bank, country risk profile of the international risk country group (IRCG), business indicators of the World Bank, and so on, with fuzzy results.[41] Instead, some economists have calibrated the role of institutions to some reduced parameters.

In our case, we also limit modelling the role of institutions by considering the effect of the institutions in the production process, represented by a parameter $\theta \geq 1$ representing the quality of institutions. For $\theta = 1$, the quality of institutions has no effect on the production function, which turns out to be the same as in the previous section. The production function then becomes:

$$Y_t = A.{K_{pt}}^{\alpha\theta\varepsilon} {H_t}^{\alpha(\theta-1)} K_{gt}^{\alpha(1-\theta\varepsilon)} L_t^{(1-\alpha\theta)} \tag{18}$$

with L_t as the quantity of labour and the other variables as previously defined.

We notice that this production function exhibits constant returns to scale[42] in all its inputs as: $(\alpha\theta\varepsilon + \alpha(\theta - 1) + \alpha(1 - \theta\varepsilon) + (1 - \alpha\theta) = 1)$. For $\theta = 1$ and $\varepsilon = 0$, the production function is a function of only public capital and labour.

Normalizing by the labour quantity L and labelling using lowercase, equation (18) yields:

$$y_t = A.{k_{pt}}^{\alpha\varepsilon\theta} {h_t}^{\alpha(\theta-1)} k_{gt}^{\alpha(1-\varepsilon\theta)} \tag{19}$$

The lowercase variables describe the per capita of the respective higher-case quantities, as defined previously. While the original production exhibits constant returns to scale, the normalized

[41] As a result of the non-convincing measures and fuzzy results of considering such measures representing the qualitative dimension of the institutions, we limit our next empirical application (Section 4) to the dimension size represented by the public capital stock, for which we add human capital, as discussed in Section 3.3.4. The quality of human capital is also questioned, as the measures considered are mainly based on the educational attainments represented generally by the average years of schooling for adults. In order to compare countries based on this measure, the main assumption is that: one year acquired in a society is the same across all other societies, neglecting by this the differences between educational systems between countries. Nevertheless, we consider that those differences are reduced, as the sample of countries enjoys a nearly equivalent level of development.

[42] The condition of constant returns to scale is crucial for an endogenous growth model.

production function's returns to scale depend on the quantity $\alpha\theta$ as the sum of the corresponding elasticities:

- $\alpha\theta = 1$: constant returns to scale. The higher the quality of institutions, the lower the input share α needed for producing
- $\alpha\theta > 1$: increasing return to scale
- $\alpha\theta < 1$: decreasing return to scale

We define ρ_x as the rate of return of the input x to the output y as the marginal rate of that input, so as: $\rho_x = \frac{\partial y}{\partial x}$. Accordingly, for public capital we have:

$$\rho_{k_g} = \frac{\partial y}{\partial k_g} = \frac{\partial Y}{\partial K_g} = \alpha(1 - \varepsilon\theta)\frac{Y}{K_g} \tag{20}$$

Similarly:

$$\rho_h = \alpha(\theta - 1)\frac{Y}{H} \tag{21}$$

And:

$$\rho_{k_p} = \alpha\varepsilon\theta\frac{Y}{k_p} \tag{22}$$

From the inventory public capital accumulation, the equation assuming the inventory stock is calculated in the beginning of the period and not in the last part of the period:[43]

$$K_{g,t} = \left(1 - \delta_{g,t-1}\right)K_{g,t-1} + G_t \tag{23}$$

where $\delta_{g,t-1}$ is the depreciation rate of public capital accumulation $K_{g,t-1}$. Dividing the equation of public capital by the first lagged output and assuming as negligible the public capital depreciation rate, we obtain:

$$\frac{\Delta K_{g,t}}{Y_t} = -\frac{\delta_{g,t-1}}{Y_t} + \frac{G_t}{Y_t} \cong \frac{G_t}{Y_t} = g \tag{24}$$

where g is government expenditure, representing the dimension size of the institutions (the state/government size). From equation (23), we have $K_{g,t} = \frac{\alpha(1-\varepsilon\theta)}{\rho_{k_g}}Y_t$, which yields:

$$\Delta K_{g,t} = \frac{\alpha(1-\varepsilon\theta)}{\rho_{k_g}}\Delta Y_t \tag{25}$$

[43] The other alternative equation is to assume that the inventory stock is assessed at the end of the period (31[th] December instead of 1[st] January), and thus the equation changes slightly to: $K_{g,t+1} = \left(1 - \delta_{g,t}\right)K_{g,t} + G_{t+1}$.

Considering (24), equation (25) is now written as $\frac{\Delta K_{g,t}}{Y_t} = \frac{\alpha(1-\varepsilon\theta)}{\rho_{k_g}} \cdot \frac{\Delta Y_t}{Y_t} = \frac{G}{Y} = g$.

As $\frac{\Delta Y_t}{Y_t} = \frac{\Delta Y_t}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} = \frac{\gamma}{1+\gamma}$, where γ is the growth rate of the economy, the dimension size yields:

$$g = \frac{\alpha(1-\varepsilon\theta)}{\rho_{k_g}} \left(\frac{\gamma}{1+\gamma}\right) \tag{26}$$

The dimension size of the state is an endogenous function of quantitative quantities related to the growth of the economy and the productivity of public capital, and to a qualitative variable representing the efficiency of human capital and the quality of government institutions. In detail:

- The dimension size of the government is a function of the economic growth rate via the term $\frac{\gamma}{1+\gamma}$: the impact of the latter depends on the sign of quantity $\frac{\alpha(1-\varepsilon\theta)}{\rho_{k_g}}$. In particular, the size is a growing function of the economic growth rate if $\varepsilon\theta < 1$ and the efficiency of the public capital is positive (the derivative is positive in this case).
- The dimension size is inversely dependent on the efficiency of public capital: the higher the productivity of public expenditure, the lower the size of the government. Alternatively, lower efficiency of public capital leads to a higher government size.
- The dimension size of the government is negatively related to the human capital efficiency and the quality of institutions. A high efficiency of human capital, as well as good quality of institutions, should lead to a small size of government.

Alternative equations for the dimension size:

$$\rho_{k_g} = \frac{\partial y}{\partial k_g} = \frac{\partial Y}{\partial K_g} = \frac{\Delta Y}{\Delta K_g} = \alpha(1-\varepsilon\theta)\frac{Y}{K_g} \tag{27}$$

$\rho_{k_g} = \alpha(1-\varepsilon\theta)\frac{Y}{K_g}$ and $g = \frac{\alpha(1-\varepsilon\theta)}{\rho_{k_g}}\left(\frac{\gamma}{1+\gamma}\right)$ yields:

$$g = \left(\frac{\gamma}{1+\gamma}\right)\frac{K_g}{Y} \tag{28}$$

Or:

$$g = \frac{\alpha(1-\varepsilon\theta)}{\frac{\Delta Y}{\Delta K_g}}\left(\frac{\gamma}{1+\gamma}\right) = \alpha(1-\varepsilon\theta)\left(\frac{\gamma}{1+\gamma}\right)\frac{\Delta K_g}{\Delta Y} \tag{29}$$

3.3.4. The government sector

The budget constraint is defined as:

$$B_t = (1+i_t)B_{t-1} + GI_t + SG_t - T_t \tag{30}$$

where B_t is government bonds, i_t is the nominal interest rate, and, GI_t, SG_t and T_t are, respectively, government flows of expenditure on productive capital, social and security government spending, and tax revenue. The quantity $GI_t + SG_t - T_t$ is the primary balance (primary surplus or deficit, depending on its sign). Some authors (Obstfeld, 1997; Haslag and Young, 1998; Terra, 2015)[44] add a term of seigniorage revenue, which we assume to be non-existent or negligible[45] in our case. We assume that state dependency on seigniorage revenue is likely to happen when the government fails to fulfil its financing needs for expenditure through conventional taxes or bond sales. Furthermore, the option of seigniorage (*fiat money*) may lead to hyperinflation, which may be self-defeating. These conditions are of less importance in advanced economies (which are the focus of the case study of this paper).

Dividing both sides of this equation by the nominal output production $p_t Y_t$ with p_t the price deflator of real output Y_t, and manipulating the left-hand-side equation term (LHS) to raise the real growth rate and inflation terms yields:

$$b_t = \frac{1+i_t}{1+\gamma_t} b_{t-1} + gi + sg_t - \tau_t \tag{31}$$

where b_t, gi_t, sg_t and τ_t are the ratios to the output of public debt, government productive investment, government social and security spending and tax revenue, respectively, while γ_t is the current nominal economic growth. This equation describes the law of motion in an inter-temporal balanced government budget.

3.3.5. The productive potential government capital and potential government debt

In this section we link the results of the productive sector (Section 3.3.1) to the government sector (Section 3.3.2). Equation (16) leads to an endogenous potential government investment (which will be simulated in the first step).

Equations (16) and (19) could be jointly used to derive a potential endogenous (limit) of public debt in relation to economic and monetary conditions (economic growth, public capital productivity, interest rate). For example, Maebayashi et al. (2017) consider that the government could have a targeted potential level of debt in the long term ($b_{t-1} = b_t = \tilde{b}_t$). We assume that this potential debt limit will be designated only to financing the potential government investment needs $(-\tilde{g}_t)$, while we can assume that social and security spending are financed by tax revenue; this means that $sg_t - \tau_t = 0$ and $gi_t = -\tilde{g}_t$ (gi_t is negative, as it is a financing need to be filled

[44] "*Seigniorage is government revenue resultant from the emission and maintenance of the stock of currency in circulation"* (Terra, 2015). "*The government resources constraint establishes that the change in government debt should be equal to the interest payment on the existing debt added to the resources necessary to supply public goods, minus the seigniorage revenue*" (Terra, 2015).

[45] Generally, money-creation revenue accounts for less than 2% of GDP (Haslag and Young, 1998).

by the new issuance of debt; otherwise, there is a positive primary surplus and accumulating new debt is not necessary). Considering this case, equation (19) yields:

$$\tilde{b}_t = \frac{(1+\gamma_t)}{(\gamma_t - i_t)}(-\tilde{g}_t) = \frac{(1+\gamma_t)}{(i_t - \gamma_t)}\tilde{g}_t \tag{32}$$

Replacing $\tilde{g}_t$, as described in (14), leads to:

$$\tilde{b}_t = \frac{(1+\gamma_t)}{(i_t - \gamma_t)}\left(\left[\frac{\gamma_t}{1+\gamma_t}\right]\frac{1}{r_{g,t}} + \left[\frac{\delta\beta}{1+\gamma_t}\right]\frac{1}{r_{g,t-1}}\right) \tag{33}$$

Equation (32) (or 33) delivers an endogenous potential (optimal/maximal/limit) debt that a country could target in the long term based on its economic fundamentals related to its potential productive expenditure, economic growth rate and interest rate. As we are considering the long term, the interest rate considered is the long-term interest rate. Note that the denominator is the difference between interest rates and economic growth. The higher this denominator, which means a higher interest rate than economic growth (unfavourable economic conditions), the lower the potential debt that a country could issue. This denominator constitutes an inertia bringing down potential debt in bad economic conditions, where interest rate spreads are higher. Inversely, the higher the growth rate, the higher the potential debt (the derivation of the potential debt to growth rate is strictly positive).

4. Empirical evidence

Considering all the previous elements and discussions, this section augments the productive sector by two additional endogenized inputs representing the government and human capital sectors. The public sector is included through its dimension size of only productive investments (gross capital formation flows and stocks).

The set of countries studied is composed of 20 advanced countries, most of which are parts of the European Union, of which 12 adhere to the European Monetary Union, while the rest of the countries generally adopt a floating exchange rate system. These countries are, respectively: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Greece, Ireland, Italy, Japan, The Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States of America.

Data for the accumulated private and public capital stocks is drawn from the IMF database,[46] for which the data is made available for 170 countries. The data range covers the period 1965–2015, and the database was last updated in January 2017. Either public or private stocks are constructed according to the inventory method following the equation:

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[46] Accessible database on excel file is through the hyperlink in the PDF document "*Investment and Capital Stock Dataset*": https://www.imf.org/external/np/fad/publicinvestment/pdf/csupdate_jan17.pdf.

$$K_{x,it+1} = (1 - \delta_{x,it})K_{x,it} + (1 - \delta_{x,it}/2)I_{x,it} \quad (34)$$

where $K_{x,it}$ is the stock of capital ($x = \{public;\ private\}$), $\delta_{x,it}$ is the corresponding depreciation rate and $I_{x,it}$ is the investment flows (gross capital formation for the sector x).

To construct the time series of the stocks using the perpetual inventory stock equation (34), assumptions made about the initial values of stock and data on investment flows are described fully in the IMF (2015).[47] Tables A.2 and A.3 in Appendix A give a summary of the data coverage, sources, description of the variables and the corresponding descriptive statistics.

Other variables such as labour and population are drawn from World Development Indicators of the World Bank database,[48] while the human capital is from the Barro and Lee website, displaying educational attainment and average years of schooling for a large set of countries and a wide range of time periods.[49] The human capital data is produced by an interval of five years from 1950 to 2010 (i.e. 1950; 1955; 1960; …; 2005 and 2010), which we extrapolated by simply moving averages to fill the gaps between the extremities of the five-year intervals. The whole methodology and the sources used to produce such data are fully explained in Barro and Lee (1993) and revised in Barro and Lee (2013).

4.1. Estimation of the Cobb-Douglas production function

The Cobb-Douglas production function encompassing human capital is modified and log-linearized to take the form of the following specification:

$$lnY_t = \alpha.\,lnK_{g,t} + \beta.\,lnK_{p,t} + (1 - \beta).\ln(h_t.\,L_t) + C + \epsilon_t \quad (35)$$

where Y_t is the output, $K_{g,t}$ represents the stock of public capital and $K_{p,t}$ the private capital stock, and L_t is the labour, adjusted for human capital by average years of schooling h_t. The private capital stock and the labour variables ensure constant returns to scale. C is a constant term and ϵ_t is the error terms assumed to behave independently and identically distributed.

We produce estimations of the previous specification using public capital stock in Table 1, over two periods of data history: the period 1960–2015; and the sub-period 1960–2007, which excludes the economic financial crisis and its subsequent impacts. Table 1 shows the estimated elasticities for all the variables of Equation (35), along with their statistical significance. Overall, the

[47] http://www.imf.org/external/np/fad/publicinvestment/data/info122216.pdf
[48] https://databank.worldbank.org/home.aspx
[49] http://www.barrolee.com/

estimations have a high significant coefficient for all countries, with high adjusted R-squared going beyond 95% for all countries and a high Fisher (F-test) global significance.[50]

The country results show that 15 out of 20 countries have significant public capital stock elastities for the two periods, with one negative significant elasticty for Greece in the first period and three negative elasticities for Austria, Greece and Japan in the second period. Over the two periods, Australia, Denmark and New Zealand have non-significant public capital elasticties. The average of accepted postive elasticties is around 0.32 and 0.36 for the two periods, respectively, and this average drops, when taking into account negative accepted elastitcities to 0.30 and 0.25, respectively, over the two periods. Higher elastictities (over 0.5) are recorded for the two periods, respectively, by Ireland (1.04 and 1.14), followed by Germany (0.59 and 0.51) and Sweden (0.57 and 0.51). Menawhile, many countries have elasticities ranging between 0.15 (the United States) and 0.25 (Canada and Belgium, for example), except France, which showed a lower accepted eslasticy value of around 0.07. Figure 2 summarizes the significant elasticities for government capital stocks, for which we assume an average of 0.30 as elasticity for the few non-significant elasticities (Australia, Austria, Denmark and New Zealand).

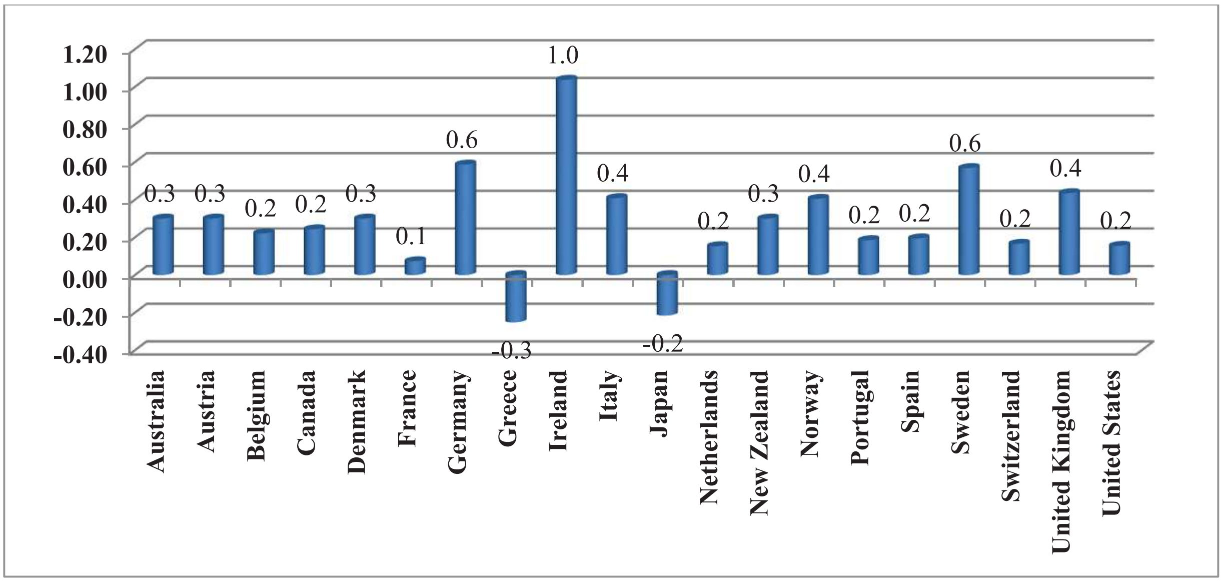
[50] The Durbin-Watson statistics are also low for all estimations, which indicates the presumption of a cointegration relationship in the data. However, in designing the rest of the simulations for the public debt potential, our model is intended to consider only the long-term effects (classical effects), while the short-term effects (Keynesian effects) are examined in Bentour (2021a), which links the effects of the public deficit financed by government bonds to fiscal policy effects via the multiplier effect (the Keynesian multiplier). Therefore, cointegration and error correction model formulations are not considered here, where we limit ourselves to long-term estimations. Besides, considering such formulations will completely modify the form of the Cobb-Douglas production function, which is theory grounded rather than ad hoc compilations, and complicate obtaining the overall elasticity effects.

Table 1. Specification estimation results using government investment stocks

		Specification: $lnY_t = \alpha . lnK_{g,t} + \beta . lnK_{p,t} + (1-\beta).\ln(h_t . L_t) + C + \epsilon_t$					
		Sample 1960-2015			Sample 1960-2007		
		Public capital K_g	*Private capital K_p*	*Intercept*	*Public capital K_g*	*Private capital K_p*	*Intercept*
		α	β	C	α	β	C
Australia	Coefficient	0.045	0.884***	-1.944*	0.007	1.090***	0.757
	Probability	0.167	0.000	0.085	0.821	0.000	0.530
Austria	Coefficient	0.207	1.239***	0.763	-0.184**	1.966***	11.002***
	Probability	0.128	0.000	0.848	0.028	0.000	0.000
Belgium	Coefficient	0.222***	0.995***	-2.097*	0.208***	0.963***	-2.415**
	Probability	0.000	0.000	0.093	0.000	0.000	0.033
Canada	Coefficient	0.243***	0.171	-11.808***	0.338***	-0.020	-14.650***
	Probability	0.000	0.113	0.000	0.000	0.860	0.000
Switzerland	Coefficient	0.166***	0.396***	-8.458***	0.145***	0.485**	-7.366***
	Probability	0.000	0.000	0.000	0.003	0.022	0.005
Denmark	Coefficient	-0.012	0.413***	-7.263***	-0.029	0.478***	-6.413***
	Probability	0.732	0.000	0.000	0.400	0.000	0.000
France	Coefficient	0.073***	0.621***	-5.514***	0.065***	0.646***	-5.173***
	Probability	0.000	0.000	0.000	0.000	0.000	0.000
United Kingdom	Coefficient	0.435***	0.390***	-10.758***	0.077	0.728***	-4.330*
	Probability	0.000	0.000	0.000	0.562	0.000	0.082
Germany	Coefficient	0.587***	0.351***	-12.470***	0.515***	0.498***	-10.256***
	Probability	0.000	0.000	0.000	0.000	0.000	0.000
Greece	Coefficient	-0.251***	2.186***	15.217***	-0.207***	2.157***	14.679***
	Probability	0.000	0.000	0.000	0.000	0.000	0.000
Ireland	Coefficient	1.038***	-1.394***	-32.801***	1.144***	-1.627***	-35.925***
	Probability	0.000	0.005	0.000	0.000	0.002	0.000
Italy	Coefficient	0.408***	1.224***	-1.167	0.428***	1.135***	-2.299***
	Probability	0.000	0.000	0.104	0.000	0.000	0.007
Japan	Coefficient	-0.215	1.075***	2.146	-0.269*	1.138***	3.336
	Probability	0.135	0.000	0.562	0.076	0.000	0.387
Netherlands	Coefficient	0.153***	1.861***	8.420**	0.107**	1.842***	8.466**
	Probability	0.003	0.000	0.012	0.040	0.000	0.015
Norway	Coefficient	0.405***	0.243***	-11.014***	0.448***	0.143***	-12.318***
	Probability	0.000	0.000	0.000	0.000	0.001	0.000
New Zealand	Coefficient	-0.009	1.552***	6.691***	-0.043	1.865***	10.733***
	Probability	0.814	0.000	0.001	0.349	0.000	0.001
Portugal	Coefficient	0.185***	0.637***	-5.795***	0.241***	0.757***	-4.674***
	Probability	0.000	0.000	0.000	0.000	0.000	0.000
Spain	Coefficient	0.195***	-0.500	-19.316***	0.185***	-0.075	-14.301***
	Probability	0.000	0.158	0.000	0.000	0.849	0.005
Sweden	Coefficient	0.568***	-0.625***	-22.670***	0.508***	-0.637***	-22.500***
	Probability	0.000	0.008	0.000	0.000	0.001	0.000
United States	Coefficient	0.155*	0.735***	-4.866*	0.083	0.863***	-2.691
	Probability	0.085	0.000	0.061	0.464	0.000	0.391

Notes: Coefficient and probability are, respectively, the estimated elasticities of the Cobb-Douglas production function (since the equation is log-linearized) and their corresponding p-values. *, ** and *** means significance at 10%, 5% and 1%, respectively.

Figure 2. Government capital stock elasticities



Source: Author's own calculations.

For estimations of Equation (35), using government capital flows instead of government capital stock, Table 2 shows that 16 out of 20 countries have significant government investment elastities for the period 1960–2015, with one negative elasticty recorded for Greece, and 17 countries for the period of 1960–2007, with two negative significant elasticties shown for Greece and Germany in this period. The average of accepted positive elasticities is around 0.24 and 0.26 for the two respective periods, while it is reduced to around 0.21 and 0.19 over the two periods, respectively, when accounting for negative accepted elasticties (Greece for the two periods, and Germany for the period 1960–2007).

By country, higher elasticities of between 0.30 and 0.50 are recorded for Sweden, Norway, Ireland, The Netherlands and Italy. Belgium and New Zealand's elasticities are, respectively, negative and non-significant and positive non-accepted for the two periods of estimations. The values of positive elasticities range between:

- Relatively low values of around 5% to 15% recorded for four countries, namely, Australia, Austria, Canada, Denmark, France, Germany and Japan;
- Medium values of around 15% to 25% obtained for countries such as Switzerland, the United Kingdom, Portugal, Spain and the United States;
- Relatively high values of public capital elasticities shown for Ireland, Italy, The Netherlands, Norway and Sweden.

Some countries tend to crowd out private capital with public capital where the elasticity of private capital is either low positive or negative accepted, as with Spain, or positive and rejected, as with Ireland (1965–2007).

Table 2. Specification estimation results using government investment flows

		Specification: $lnY_t = \alpha.lnI_{g,t} + \beta.lnK_{p,t} + (1-\beta).\ln(h_t.L_t) + C + \epsilon_t$					
		1960-2015			1960-2007		
		I_g	K_p	*Intercept*	I_g	K_p	*Intercept*
		α	β	C	α	β	C
Australia	Coefficient	0.115***	0.667***	-4.604***	0.119***	0.778***	-3.258**
	Probability	0.002	0.000	0.001	0.001	0.000	0.011
Austria	Coefficient	0.143**	1.543***	4.953***	0.004	1.575***	5.615***
	Probability	0.013	0.000	0.000	0.918	0.000	0.000
Belgium	Coefficient	-0.009	1.978***	10.167***	-0.030	1.931***	9.671***
	Probability	0.894	0.000	0.003	0.639	0.000	0.005
Canada	Coefficient	0.120***	0.500***	-6.801***	0.210***	0.340***	-8.999***
	Probability	0.000	0.000	0.000	0.000	0.000	0.000
Switzerland	Coefficient	0.236***	0.396***	-8.165***	0.181***	0.567***	-6.149***
	Probability	0.000	0.000	0.000	0.003	0.003	0.005
Denmark	Coefficient	0.019	0.421***	-7.279***	0.044**	0.528***	-6.080***
	Probability	0.233	0.000	0.000	0.012	0.000	0.000
France	Coefficient	0.106***	0.634***	-5.300***	0.099***	0.656***	-5.017***
	Probability	0.000	0.000	0.000	0.000	0.000	0.000
Great Britain	Coefficient	0.183***	0.805***	-3.567***	0.102***	0.810***	-3.215***
	Probability	0.000	0.000	0.000	0.010	0.000	0.000
Germany	Coefficient	0.142	0.853***	-3.005	-0.273**	1.401***	5.042**
	Probability	0.366	0.000	0.210	0.048	0.000	0.025
Greece	Coefficient	-0.109***	1.776***	9.299***	-0.130***	2.018***	12.317***
	Probability	0.004	0.000	0.000	0.000	0.000	0.000
Ireland	Coefficient	0.302***	0.591***	-5.965**	0.419***	0.196	-10.743***
	Probability	0.000	0.004	0.015	0.000	0.387	0.000
Italy	Coefficient	0.472***	0.943***	-3.415***	0.479***	1.246***	-0.047
	Probability	0.000	0.000	0.008	0.000	0.000	0.965
Japan	Coefficient	0.077**	0.684***	-4.721***	0.211***	0.513***	-7.527***
	Probability	0.049	0.000	0.000	0.000	0.000	0.000
Netherlands	Coefficient	0.321***	1.082***	-0.668	0.292***	1.136***	0.045
	Probability	0.000	0.000	0.791	0.000	0.000	0.985
Norway	Coefficient	0.398***	0.213***	-10.281***	0.467***	0.044	-12.289***
	Probability	0.000	0.006	0.000	0.000	0.516	0.000
New Zealand	Coefficient	0.042	1.363***	4.256*	0.028	1.562***	6.754*
	Probability	0.335	0.000	0.074	0.552	0.000	0.052
Portugal	Coefficient	0.180***	0.624***	-5.421***	0.212***	0.744***	-4.128***
	Probability	0.000	0.000	0.000	0.000	0.000	0.001
Spain	Coefficient	0.163***	-0.239	-15.649***	0.159***	-0.027	-13.159***
	Probability	0.000	0.384	0.000	0.001	0.948	0.010
Sweden	Coefficient	0.520***	0.337	-9.695***	0.427***	0.383	-8.944***
	Probability	0.000	0.158	0.001	0.000	0.110	0.003
United States	Coefficient	0.193***	0.624***	-5.974***	0.209***	0.602***	-6.340***
	Probability	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Coefficient and probability are, respectively, the estimated elasticities of Cobb-Douglas production function (since the equation is log-linearized) and their corresponding p-values. *, ** and *** means significance at 10%, 5% and 1%, respectively.

The overall average of accepted elasticities over the two periods is around 20%. This value is somewhat below the average value, which we estimated using panel data for the whole sample of the same countries. The estimated value for the panel group is around 25% and corresponds exactly to what is found by Barro's (1993) panel estimations for government investment flows. Focusing on the United States' data, the elasticity is around 0.19 and 0.21 over the two periods and is very robust to data sample variations in our estimations.

A conducted robustness check generally shows that the elasticity of public capital flows for the United States varies between 17% and 0.23% following sample time variations. Furthermore, Hill (2008) shows that the growth-maximizing size of the state varies between 9% and 29% for the United States (large interval), while Scully (1996), with a different specification to ours, reports a government size of 19% for the same country. For the countries for which government investment flow elasticities are rejected over the two periods of estimation, we can consider the average of all elasticities (0.20) as a calibrated elasticity for all these countries in the subsequent calculations. We can also adopt calibrated parameters based on similarities between countries. For the case of Japan, however, an estimation over earlier samples gives highly accepted positive but decreasing elasticities over time. Belgium also has a sensitive elasticity to sample changes. The elasticity becomes accepted starting from 1980, where we find a positive accepted elasticity of 7.6% over 1980–2015.

Inspecting the elasticities of public and private capital, we notice that higher government capital elastictities (and also higher government flow elasticities) are accompanied by lower or negative private capital (flow) elasticities or rejected private capital elasticties. This is particularly the case for countries such as Ireland, Norway, Spain and Sweden. This might be attributed to the crowding-out effect of private investment by government investment. Drawing scatter plots for private and public capital elasticities for the two periods clearly shows this tendancy. Figures 3 and 4 present a negative relatiship between public and private capital elasticties for the periods 1960–2015 and 1960–2007, respectively, with Ireland and Greece representing two opposite extreme points in these figures (the red-coloured dot in the two figures indicates the simple average of elasticties of the sample). The same trend is shown when considering investment flows instead of stocks (Figures B.1 and B.2 in the appendix).

Figure 3 : Scatter plot elasticities of Public Investment stocks versus Private capital stocks (*Kp, Ig*): Sample 1960-2007

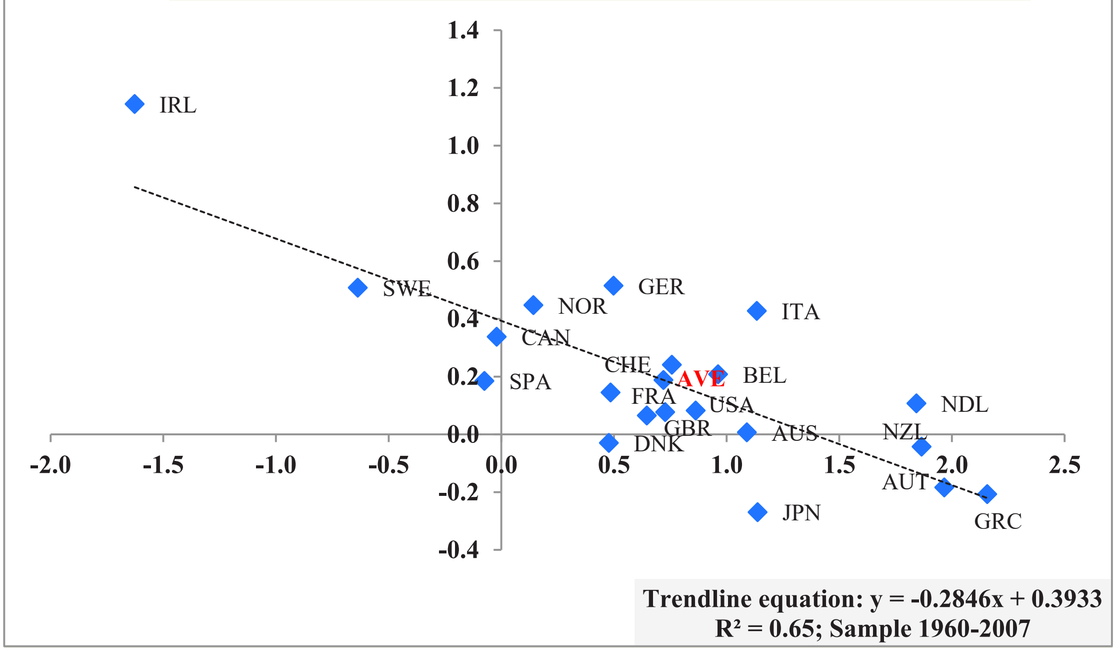
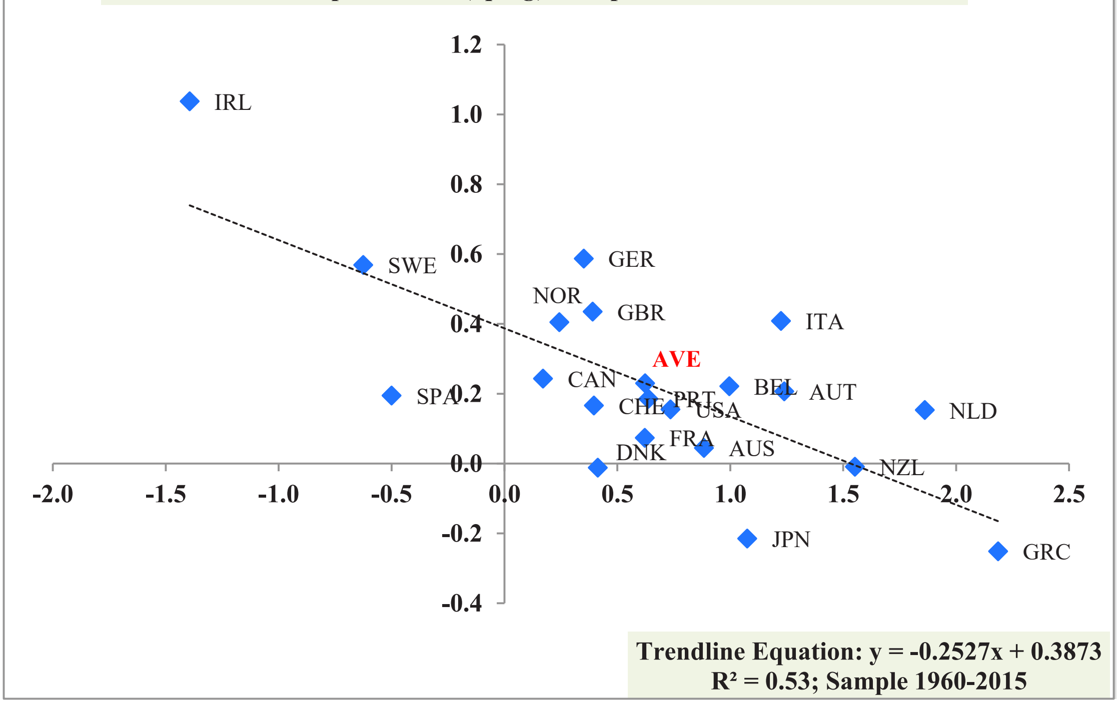


Figure 4 : Scatter plot elasticities of Public Investment Stocks versus Private capital stocks (*Kp, Ig*): Sample 1960-2015



4.2. Simulations of targeted/potential public debt ratios

Based on the results of Table 1 for capital stock elasticities, we note that there are no big differences between elasticities for the two periods (1960–2007 and 1960–2015). Therefore, to save space, we pursue our calculations considering the results of elasticities over the whole period of 1960–2015. To conduct simulations and calculus, we decided to produce two simulated exercises. The first one is based on the estimated elasticity of each country, and the second is based on the average of all countries' elasticities (equal to 0.3).

However, for the countries for which elasticities are not accepted across the simulated period, we decided to consider only the average of elasticities as the benchmark for their elasticities. These countries are Australia, Austria, Denmark and New Zealand. Greece, over both periods, and Japan, over the period 1960–2007, although they have negative accepted elasticities, are analysed assuming these are the true elasticities. Hence, the simulation is done considering their negative elasticities[51] and the average of the elasticities (0.30). For the rest of the countries, we simulate the rest of our calculations considering the countries' elasticities and the average of countries' elasticities (0.3).

Simulation steps

- First, we use the previous government capital values' elasticities (Table 1), estimated for the period 1965–2015, to generate the "return" of the government productive investment using the formulae $r_{g,t} = \frac{\partial Y}{\partial K_g} = \alpha \frac{Y}{K_g}$ (equation (12)).
- Second, we generate the public potential investment based on equation (15) $\left(\tilde{g}_t = \left[\frac{\gamma_t}{1+\gamma_t}\right]\frac{1}{r_{g,t}} + \left[\frac{\delta\beta}{1+\gamma_t}\right]\frac{1}{r_{g,t-1}}\right)$.
- To avoid high fluctuations generated by data, we use the Hodrick-Prescott filter[52] to smooth the generated potential public investment, before introducing it in the next step. Focusing

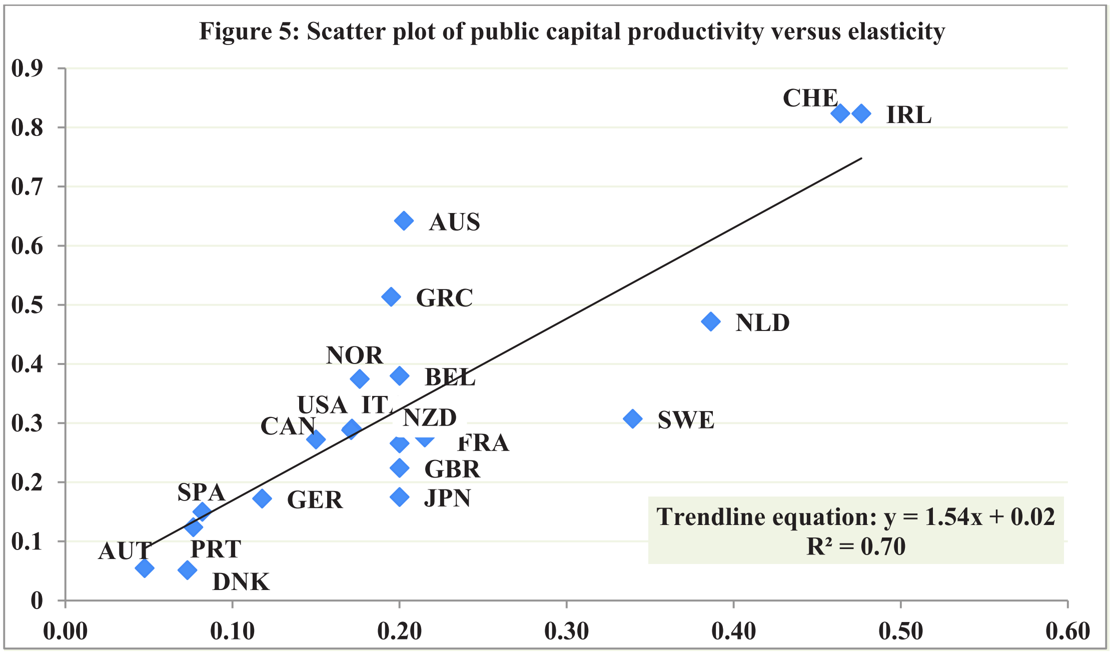
[51] Having negative elasticities does not mean that this should lead to negative simulated investment. These two countries have positive simulated investment (see Table 2 for elasticities and Figure B.3 for simulated public investment flows).

[52] To obtain a smooth estimate of the long-term trend component of a series, the Hodrick-Prescott filter (HP filter hereafter) is a widely used smoothing method among researchers. The method first appeared in a working paper in the early 1980s, was applied to analyse the post-war US business cycles and published later in 1997 (Hodrick and Prescott, 1997). A time series Y_t could be decomposed to its long-term trend G_t (a sum of growth component) and cyclical component C_t: $Y_t = G_t + C_t$. The HP filter algorithm works to smooth the original series by estimating its trend component, while the cyclical component results as the difference between the original series and its trend. The trend component is the one that minimizes $\sum_1^T (C_t)^2 + \lambda \sum_1^T [(G_t - G_{t-1}) - (G_{t-1} - G_{t-2})]^2$, where T is the number of observations and λ is a positive parameter of smoothing that depends on the frequency of the time series. The higher the data frequencies, the larger the value of λ, and the larger λ, the higher the penalty of changes in the trend's growth rate (represented by the second term of the previous equation) and the smoother the trend component. In practice, λ is set empirically to be 1,600 for quarterly data, as suggested by Hodrick and Prescott (1997), while for annual data, λ is set to 100 in many applications, which we also consider in ours.

on the long-term tendency and avoiding fluctuations seems to be in line with our focus on the long-term analysis.

- Third, we simulate the potential public debt using equation (32) $\left(\tilde{b}_t = \frac{(1+\gamma_t)}{(i_t-\gamma_t)}\tilde{g}_t\right)$.

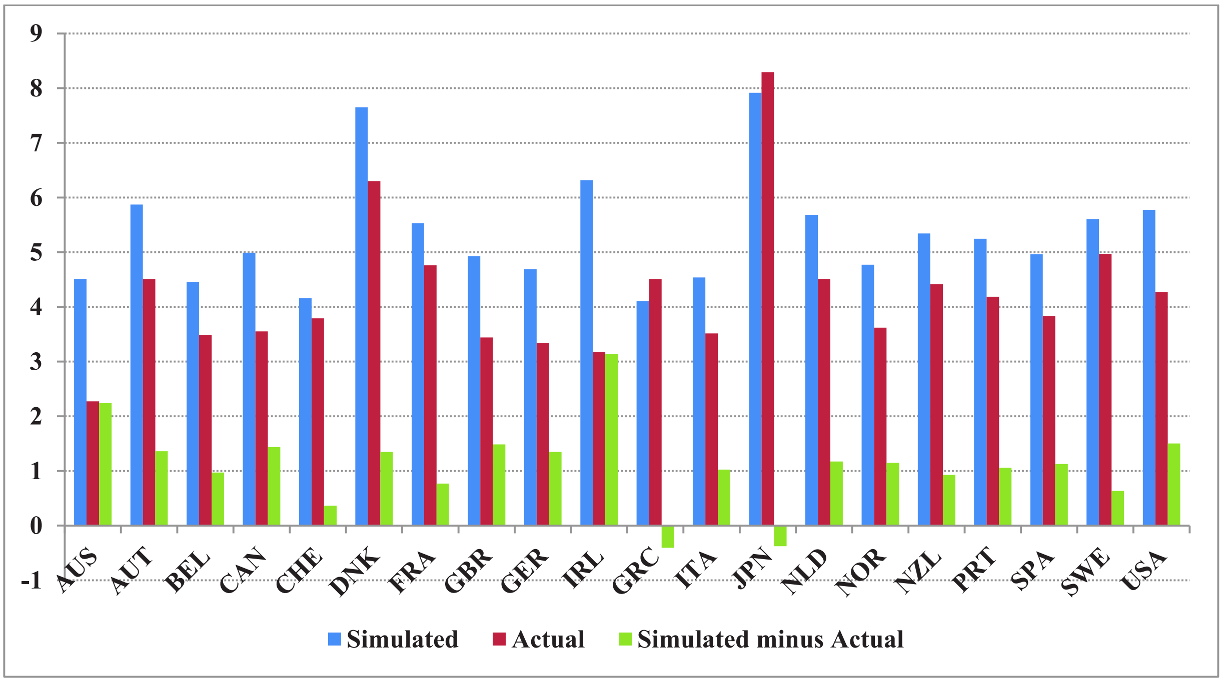
First, generating public capital returns/productivity shows, on average, big differences between countries due, mainly, to differences in elasticities. The public capital productivity averages are widely different across countries, ranging from low values of 5% to 15% in Austria, Denmark, Portugal and Spain, and very high values of 80% recorded in countries with high values of elasticities, namely, Ireland and Switzerland (Figure 5).



Second, we simulate the potential government investment based on the public capital marginal productivity (derived from the marginal product of capital, equation (15)). For comparison, we produce the descriptive statistics for both observed and simulated government productive investment (gross capital formation) and draw the averages in Figure 6. This shows that, overall, the simulated variable overcomes the actual one in all countries by about one to three percentage points, except for Japan and Greece, where the actual one is, on average, slightly above the simulated one (by 0.4 percentage points). In particular, the simulated debt ratio is double the actual one in Australia and Ireland. Figure B.3 in Appendix B shows the tendency for all countries towards actual and simulated potential government investment.[53] In all countries, government investments (actual and simulated) tend to decrease over time.

[53] For all figures and tables when it applies, we point to variables in the form of "Y_XXX", where Y is the variable presented (simulated) and XXX is the three-character country code. Table A.4 (in the appendix) presents these codes.

Figure 6. Actual and simulated productive public investment flows (gross capital formation) as % of GDP



Third, we simulate the results for the debt limit that a country could target, considering the previously simulated potential government investment and the conditions of economic and monetary performance, reflected by the average long-term interest rates and economic growth, respectively. Data on long-term interest rates is extracted from the OECD database.[54] We call this simulated debt "potential debt". We use the term "potential", as this is related to "potential" government investment (potential, as it is derived from the first condition of maximizing output, that is, marginal productivity of capital). It is, in fact, the optimal (maximal) public debt that a country can issue to finance its potential government productive investment and is directly related to public capital productivity (efficiency). This potential public debt is like a mirror (an opposite picture) of the actual debt. This is due to the differential between interest rates and economic growth that appears in the denominator of the potential debt with an opposite sign of the actual accumulated debt formula. This constitutes an inertia lowering potential debt when interest rates are higher than economic growth, which pushes down potential debt under actual debt, attracting the attention of policy-makers to the danger of the debt situation. The danger of public debt could be measured by the distance between simulated and actual debt. The higher this distance, the safer the debt is.

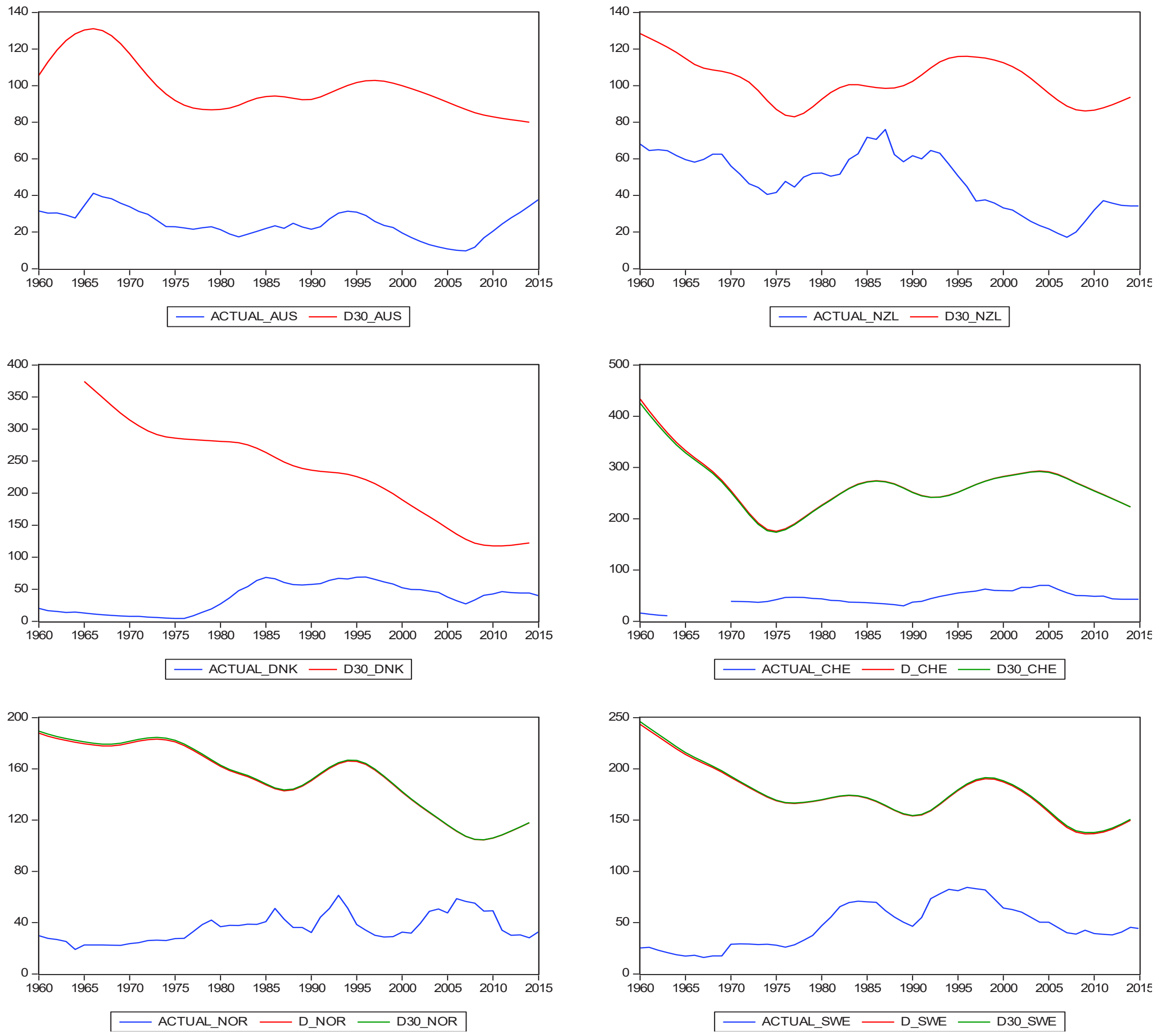
First, we consider the simulation using the growth and long-term interest rate differential average over the period 1960–2015. We produce a set of figures that show the tendencies of simulated

[54] https://stats.oecd.org/

public debt for each country. Figures 7, 8, 9 and 10 produce such simulated debt results, along with actual public debt, over the period 1960–2015 for the 20 sample countries.

The first set of graphs (Figure 7) presents six countries where the debt is not threatening, as we could still target potential debt that is higher than actual debt over history. These countries are Australia, New Zealand, Denmark, Switzerland, Norway and Sweden. The average potential debt to GDP is approximately 96.9% for Australia, and 100.5% for New Zealand, with lower values of around 80% for both countries. The remaining countries have higher potential average of public debt based on their performances, which are around 230% in Denmark, 250% in Switzerland, 150% in Norway and 170% in Sweden, respectively.

Figure 7. Simulated potential paths of public debt versus the actual path based on potential simulated government investment and economic and monetary performances for Australia, New Zealand, Denmark, Norway, Sweden and Switzerland*



* Countries having only two curves are those for which elasticities aren’t significant, and thus we limit the exercise of simulated public debt to their elasticities as the average of all significant elasticities (0.3).

Figure 8 shows a group of four non-euro area countries with floating exchange rates and larger-sized economies, namely, the United States, Japan, the United Kingdom and Canada. The sustainable path of the first three countries is undermined during the financial crisis. Actual public debt remains with a long history under targeted public debt until around the period of crisis (2008–10), although the two paths (simulated and observed) converge over time, showing that these countries accumulate public debt over time during the prosperity periods of the 1960s and the Great Moderation Era of the 1990s. For Japan, the formula shows negative explosive potential debt based on the average long-term interest rate and growth rate over the whole period. However, for this country, the long-term interest rate is observed over 1989–2015 only. A robustness check for countries is conducted based on their performances over certain specific periods and not the whole period. However, simulation is always produced over the whole period to visualize the track of simulated debt over the entire period.

For the other countries, the United States and Canada, although potential debt has a tendency to decrease over time to converge towards actual increasing debt, it still does not constitute a threat to public finance sustainability. However, the United Kingdom presents a different picture compared to the United States and Canada. Simulated debt is exceeded by actual debt in the year 2008 and stays under actual debt for the period 2008–15, despite an upward shift. The average potential debt ratios to target to finance potential expenditure is around 195.5% for the United States, 100.2% for the United Kingdom and 145.5% for Canada.

Figure 8. Simulated potential paths of public debt versus the actual path based on potential simulated government investment and economic and monetary performances for the United States, Japan, the United Kingdom and Canada

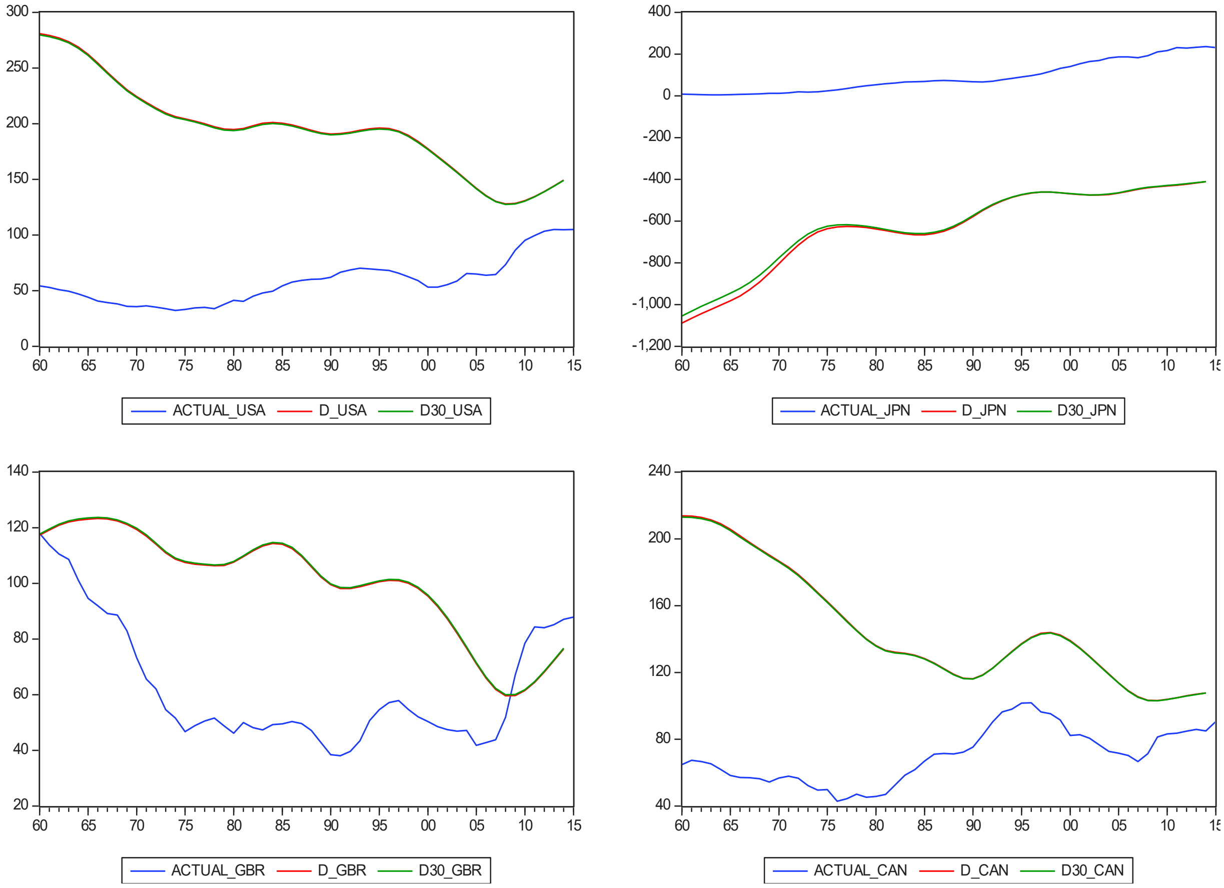
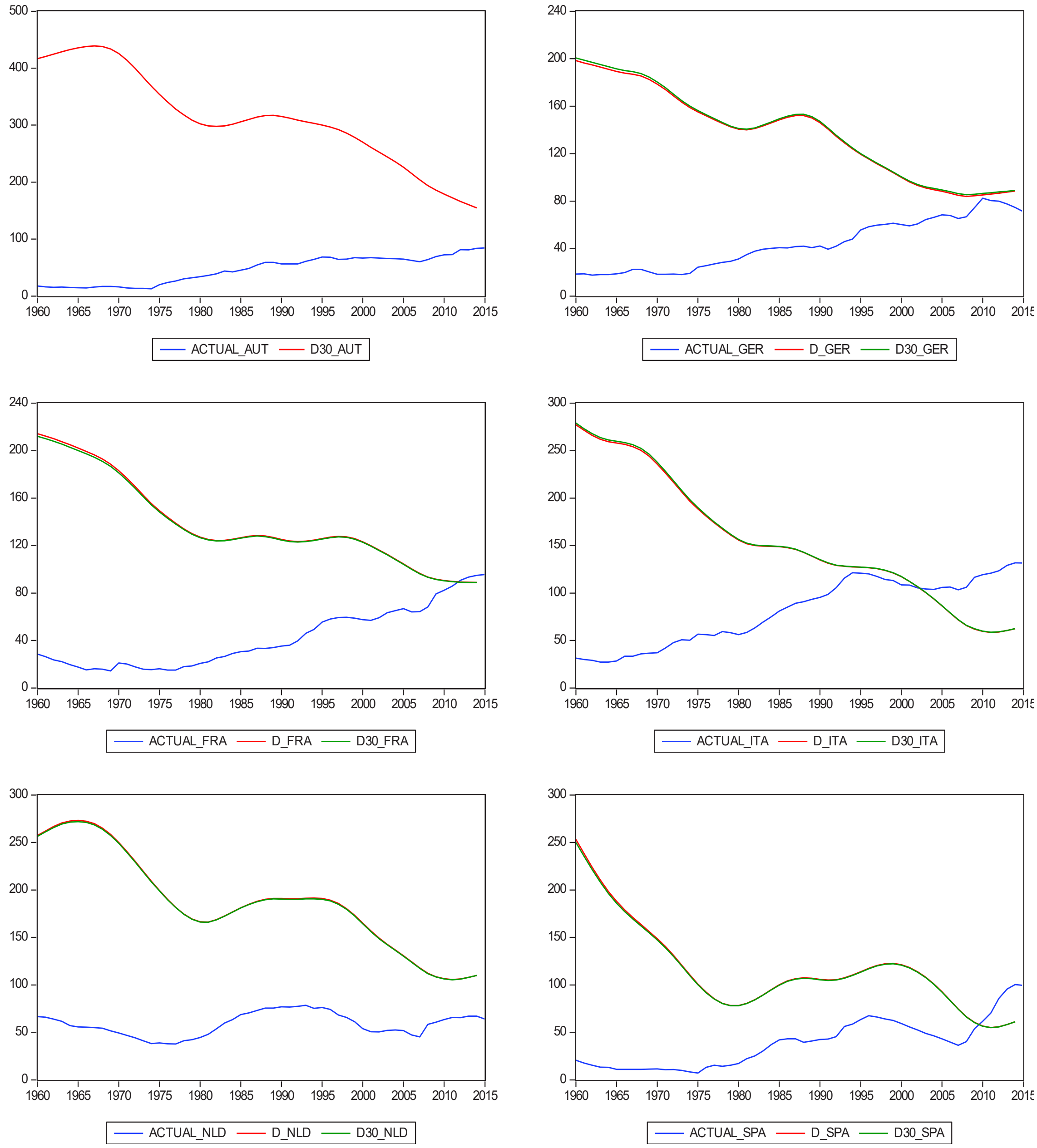


Figure 9 presents the results for a set of euro area countries, namely, Austria, Germany, France, Italy, The Netherlands and Spain.[55] For Austria and The Netherlands, potential debt remains higher than actual debt over the whole period. However, for Italy, Spain and, to some extent, France, potential debt falls below actual debt in 2002 for Italy (with potential equalling actual around 100%), in 2012 for France (with equality in 91%) and in 2010 for Spain (with equality in around 60%). Germany also has a negative trend of potential debt approaching actual debt in around 2010 (at nearly 90%) without crossing it. Potential debt stays below actual debt at around 55% in Spain, 58% in Italy and 88% in France. Therefore, these could be the safer limits of debt (not altering growth) for these countries, and they could keep their public debt under control at lower ratios than these values. In conclusion, for this set of countries, the potential debt to target in bad times is

[55] As a reminder, note that Austria's estimated elasticity is not statistically significant, and the graph shows the simulated debt for Austria based on the average of elasticities equalling 0.3.

around 60% to 90%, based on an average of their economic and monetary performances. However, in normal economic conditions, potential debt could reach more than double these values.

Figure 9. Simulated potential paths of public debt versus the actual path based on potential simulated government investment and economic and monetary performances for Austria, Germany, France, Italy, The Netherlands and Spain*



*: Countries with only two curves are those for which elasticities are not significant, and thus we limit the exercise of simulated public debt to their elasticities as the average of all significant elasticities (0.3)

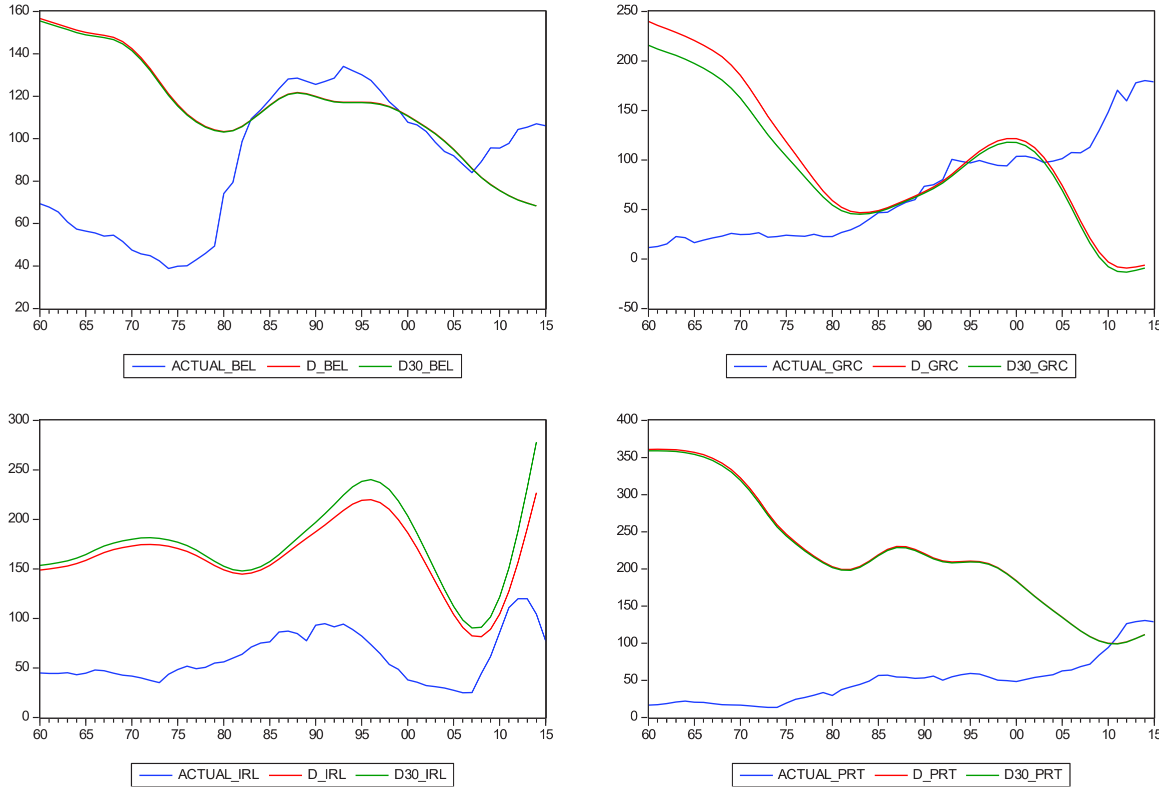
The last group of countries (Figure 10) focuses on the rest of the eurozone countries, namely, Greece, Portugal, Ireland and Belgium. Except for Portugal, which has the same tendency for potential debt as Spain and Italy (Spain and Italy are described in Figure 9), the three other countries have different stories of their own. Belgium's potential debt goes below its actual debt in the early years of the 1980s and follows the same tendency as actual debt until 2008, where it drops and the two curves disconnect and evolve oppositely. Potential debt continues to decrease until it reaches a lower value of 68% at the end of the period.

For Greece, potential debt crosses actual debt in 1985 at a lower rate of public debt around 47% and remains with the trend of actual debt for a decade, where potential debt once again surpasses actual debt in 1995 and stays until 2002, where it drops again below actual debt (as in Italy). Potential debt continues to decrease under actual debt until the end of the period and even becomes negative starting in 2010, reaching around -10% in 2011–15. The negative number should be interpreted in the sense that Greece in this period should not hold any public debt based on its economic performance at this time.

Nevertheless, some results create serious questions about the validity of the data of interest rates, inflation, growth, and so on, used to simulate the formulae for those countries and their sensitivity to the actual data.[56] For Ireland, there are two periods where debt is unsustainable (targeted debt is below actual debt), in 1983–93 and 2008–14. However, Belgium shows an apparently unsustainable path from the 1980s.

[56] Data distribution is relatively heterogenous between countries, where some variables are normally distributed for some countries and others are not, as shown by Jarque-Bera, Skewness and Kurtosis for this data in Table A.3 (Appendix A).

Figure 10. Simulated potential paths of public debt versus the actual path based on potential simulated government investment and economic and monetary performances for Belgium, Greece, Ireland and Portugal



4.3. Robustness check

- *The impact of data shortness*

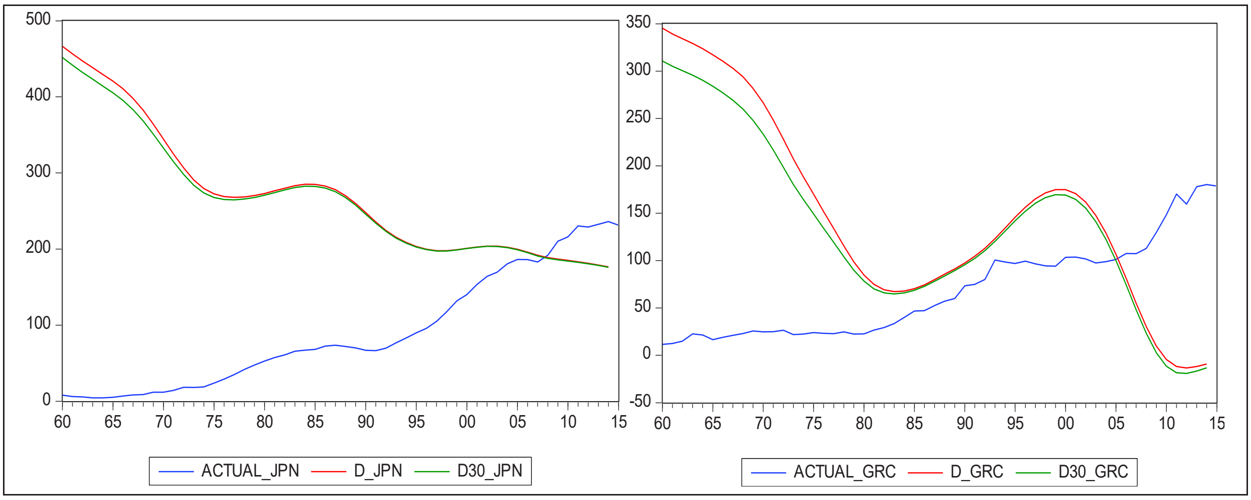
The data on long-term interest rates from the OECD database is unfortunately short samples for some countries such as Japan, where the data time series starts in 1989, and for Greece in 1998.[57] Therefore, for robustness check and to avoid the problem of data shortness, we produce estimations for potential debt, calibrating the differential of growth and interest rate in the denominator of Equation (32) by the data of the United States (but we leave the growth rate in the numerator unchangeable and proper to the country itself). This is also a test for our formula against some irregular observations in the data. In fact, countries such as Japan, Greece and Ireland are interesting cases to study, with higher accuracy and deep investigation of the data, and even

[57] For this purpose we checked many other international sources reporting data on interest rates for government bonds, securities and treasury bills, such as the International Financial Statistics (IFS) of the IMF, the Bank for International Settlements (BIS) and the Federal Reserve Bank of Saint Louis (https://fred.stlouisfed.org/tags/series?t=interest+rate%3Blong-term). These sources report data on long-term interest rates for different periods, depending on the country, which is the same data as reported by the OECD database.

specifications to be re-estimated. Along the same lines, Barta (2018) analyses and compares the cases of Belgium, Canada, Denmark, Greece, Ireland, Italy and Japan, since the 1970s, to identify factors that differentiate countries accumulating threatening debt from those that keep their debt under control. He notes that practices in fiscal policy management make a difference rather than the political bias impact of debt accumulation, as proposed by some authors in reference to the positive approach of public debt (Persson and Svensson, 1989; Alesina and Tabellini, 1990).

Figure 11 shows the results for Greece and Japan for this exercise. According to the new simulated debt for Japan, this becomes positive and high with a downward tendency until crossing the increasing accumulated debt in 2008 at the ratio of 192%. The results produced for Greece are also improved compared to its own data on interest rates, as previously explained. The intersection between actual and potential debt is materialized in 2005 at an average ratio of debt equivalent to 100%. Indeed, an IMF note on fiscal space calculating the debt limit based on the fiscal reaction function, and the interest rate growth differential, shows that the public debt dynamics are not on a sustainable path to converge to a finite steady-state ratio for the following countries: Greece, Italy, Japan and Portugal (Ostry et al., 2010).

Figure 11. Simulated potential public debt for Greece and Japan using the US long-term interest rate and economic growth differential



An exercise of simulation is also conducted for all the other countries calibrating the gap (long-term interest rate – growth) by the United States one, but although the tendency changes slightly for many countries, the years of intersections between actual and simulated debt are delayed for some countries, such as Belgium, until the year of the financial crisis (the intersection using its own data is around 1982). For the United Kingdom, potential debt stays higher even in times of crisis when calibrating with the United States' interest rates, and for many other countries the average of the simulated debt increases substantially, especially for Australia, Austria, New

Zealand, Belgium, France, Italy, Spain, Greece and The Netherlands. However, for some countries, such as Denmark, Sweden, Norway and The Netherlands, the average is almost stable between the two exercises, while it decreases substantially for Switzerland. The results for all the sample countries are stacked in Figure B.4 in Appendix B.

- *The impact of elasticity*

To gauge the effects of elasticity changes, we assess the effects of three values – the estimated one from the model, then the one calibrated to 0.15 and 0.30 values – on potential government investment and debt (Table 3). First, we see that the impact of variation of elasticities is higher for small countries than sizeable countries. Doubling the value of elasticity (for example, from 0.15 to 0.30) leads to a decrease in simulated public debt, on average, by around three points of GDP for small countries such as Greece and Ireland. This impact is, however, contained, on average, at around 1 to 2 points of GDP for sizeable countries (the United States, Japan, Germany, France, the United Kingdom).

Table 3. The effects of elasticity changes on the potential public debt limit

	D		D15		D30		D15-D30		D-D30	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Australia	99.3	96.2	98.3	95.2	96.9	93.9	1.3	1.4	2.3	2.4
Austria	300.7	303.0	301.1	303.3	300.2	302.6	0.9	0.7	0.6	0.4
Belgium	111.4	114.3	111.7	114.5	111.0	114.0	0.7	0.4	0.4	0.2
Canada	138.9	132.3	139.4	132.8	138.5	132.0	0.9	0.9	0.3	0.3
Denmark	233.3	237.7	233.0	237.5	232.7	237.2	0.3	0.2	0.6	0.4
France	131.1	125.8	130.9	125.7	130.4	125.4	0.4	0.3	0.7	0.4
Germany	130.9	140.6	132.6	141.8	132.0	141.4	0.6	0.4	-1.1	-0.8
Greece	90.1	82.9	84.5	80.7	82.6	79.8	1.9	0.9	7.6	3.1
Ireland	163.5	167.3	177.9	176.2	175.1	174.8	2.8	1.4	-11.6	-7.5
Italy	143.3	136.8	145.0	137.5	144.0	137.1	1.0	0.4	-0.7	-0.3
Japan	-591.1	-592.7	-585.1	-589.2	-582.8	-587.8	-2.3	-1.4	-8.2	-5.0
Netherlands	178.9	181.5	178.9	181.5	178.3	181.1	0.6	0.4	0.6	0.4
New Zealand	101.7	100.8	101.0	100.4	100.4	100.0	0.6	0.4	1.3	0.8
Norway	150.5	154.8	152.4	156.8	151.3	155.6	1.2	1.2	-0.8	-0.8
Portugal	209.8	209.8	210.2	210.1	208.5	208.8	1.8	1.3	1.4	1.0
Spain	105.1	105.7	105.4	105.8	104.6	105.2	0.8	0.6	0.6	0.4
Sweden	170.3	170.5	171.7	171.2	171.2	171.0	0.5	0.2	-0.9	-0.4
Switzerland	254.7	259.6	254.8	259.6	253.4	258.9	1.4	0.8	1.3	0.7
United Kingdom	98.1	101.6	98.9	102.4	98.5	102.0	0.4	0.4	-0.4	-0.4
United States	187.4	194.9	187.5	194.9	186.7	194.0	0.8	0.9	0.7	0.9

- *The impact of the differential "long-term interest rate – economic growth rate"*

Since the elasticity impact is not highly determinant on the results, as previously shown, we keep constant the elasticity of public capital at the average of 0.30[58] for all countries and simulate the impacts of the gap between interest rate and growth. In fact, as shown in the previous paragraph, the elasticity impact is very low on the results compared to what we will show for the differential in interest rates and economic growth. Furthermore, we focus on some countries where long-term interest rate data is available over the whole history. We choose, for example, sizeable countries, namely, the United States of America, France, the United Kingdom and Canada (Japan and Germany were excluded for reasons of data shortness on interest rates). We simulate results considering the whole period, and the two sub-periods 1960–84, as well as in the so-called Great Moderation Era,[59] 1985–2015, to see the effects of the gap between interest rates and growth (the denominator in Equations 30 or 31). Descriptive statistics of long-term interest rates and economic growth for these countries are displayed in Table 4. We note that the difference between long-term interest rates and economic growth is higher in the period 1960–84 than in the period 1985–2015.

Table 4. Real long-term interest rates and real economic growth differential for selected countries

	Long Term Interest Rates (% per annum)		GDP Growth rate (%)		Gap [interest rate-growth]	
Sample: 1960-2015	Mean	Median	Mean	Median	Mean	Median
France	7.3	6.7	2.9	2.4	4.5	4.3
United States	6.3	6.1	3.1	3.3	3.2	2.8
United Kingdom	7.8	7.5	2.4	2.6	5.4	4.9
Canada	7.0	6.8	3.2	3.1	3.8	3.7
Sample: 1960-1984	Mean	Median	Mean	Median	Mean	Median
France	9.4	8.6	4.2	4.5	**5.2**	4.0
United States	7.3	6.8	3.6	4.5	**3.7**	2.4
United Kingdom	9.7	8.8	2.5	2.7	**7.2**	6.1
Canada	8.3	7.6	4.2	4.0	**4.1**	3.5
Sample: 1985-2015	Mean	Median	Mean	Median	Mean	Median
France	5.6	4.9	1.8	2.0	**3.8**	2.9
United States	5.5	5.3	2.7	2.7	**2.8**	2.5
United Kingdom	6.2	5.1	2.3	2.5	**3.9**	2.5
Canada	6.0	5.5	2.4	2.6	**3.5**	2.9

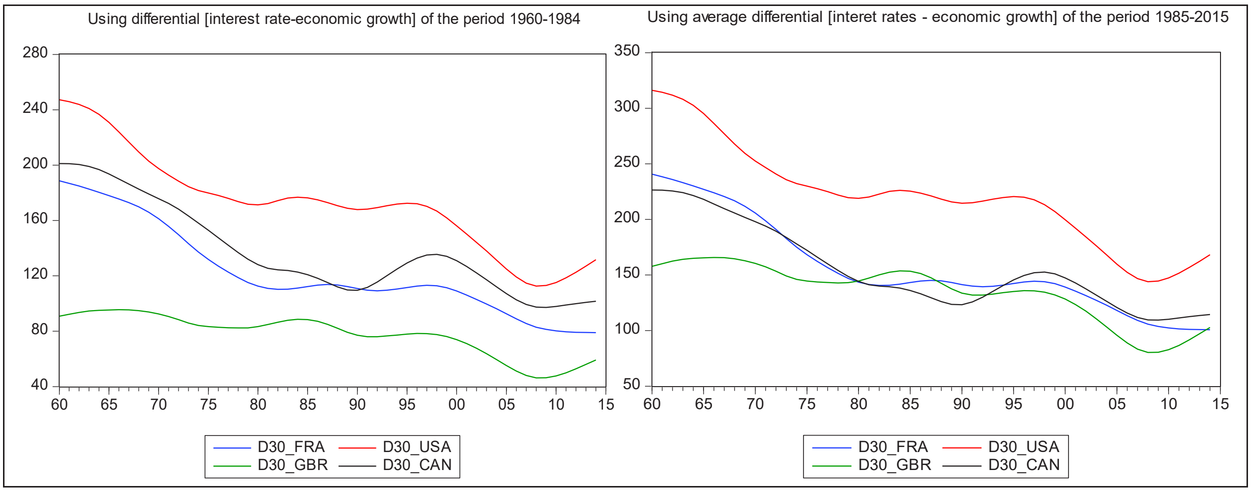
[58] We should remember that this is the average of the significant estimated public capital elasticities over the sample of countries. Fixing the elasticity for all countries to a common value allows comparison of the countries' results based on the impact of the interest rate and economic growth gaps alone.
[59] The Great Moderation Era is first described by Stock and Watson (2003) analysing, over the period 1960–2002, the United States quarterly GDP volatility, shown to be highly reduced after 1985 compared to the previous period. This tendency of output and price stability is mainly attributed to the macroeconomic stabilization policies followed by independent central banks (Bernanke, 2004). Some economists argue that this period came to an end in 2007 with the 2008 financial crisis, while others argue that the GDP volatility averages are still lower than pre-1985, assuring the continuity of the Great Moderation Era (Clark, 2009).

The effects of interest rates and economic growth differences are produced in Table 5 and Figure 11. Overall, there are sizeable and substantial effects on potential debt for the four selected countries. The averages of simulated public debt, over the period 1960–84 (Scenario 1), is around 168% for the United States, 118% for France, 78% for the United Kingdom and 135% for Canada. These averages are, respectively, higher in the Great Moderation Era, 1985–2015 (Scenario 2), by around 38 points of GDP in France, 52 points in the United States, 57 points in the United Kingdom and 20 points in Canada. For the whole period, the averages of simulated debt are 137% in France, 195% in the United Sates, 100% for the United Kingdom and 145% for Canada.

Table 5. Simulated effect of differences between long-term interest rates and economic growth

Simulating potential debt over the period 1960-2015, using constant elasticity of public capital = 0.30				
	France	United States	United Kingdom	Canada
Mean	137.4	194.7	100.5	145.1
Median	126.2	194.6	106.6	134.3
Maximum	212.1	279.7	123.7	213.0
Minimum	88.8	127.4	59.9	102.9
Scenario 1: simulating potential debt using interest rates and growth of the period 1960-1984				
	France	United States	United Kingdom	Canada
Mean	122.2	172.2	77.6	137.1
Median	112.3	172.1	82.3	126.9
Maximum	188.6	247.3	95.5	201.2
Minimum	79.0	112.6	46.3	97.2
Scenario 2: simulating potential debt using interest rates and growth of the period 1985-2015				
1985-2015	France	United States	United Kingdom	Canada
Mean	156.0	220.1	134.7	154.4
Median	143.2	220.0	142.9	142.8
Maximum	240.7	316.1	165.8	226.5
Minimum	100.8	144.0	80.3	109.5
Scenario 2 - Scenario 1				
1985-2015	France	United States	United Kingdom	Canada
Mean	33.7	47.9	57.1	17.3
Median	31.0	47.9	60.5	16.0
Maximum	52.0	68.8	70.2	25.3
Minimum	21.8	31.4	34.0	12.2

Figure 12. Simulated potential debt sensitivity to the differential of interest rates and economic growth for selected countries



4.4. Discussion

Contrary to researches adopting a short-term approach assessing the impact of the presence of public debt on GDP via the effects on the fiscal multipliers, as well as the interaction with the business cycle and other economic variables (Bentour, 2021a), this paper adopted a long-term approach based on an endogenous growth model to assess the relationship between public debt and economic growth. Economists (classical and new classical) adopting this approach generally try to avoid the role of empirical estimations and data in their calculations (adhering to the Lucas (1976) and Sims (1972) critiques). They prefer calibration to estimation and argue as if all economies are in the steady state in the long term, which is not necessarily true. Furthermore, calibration is another way of deriving parameters that are under the control of economic theory and do not necessarily reflect the behaviour and data-generating process proper to each country. Furthermore, the results of the calibrated models are theoretical and not so easy to interpret by the policy-makers. In our case, we judge it useful that countries are not necessarily in the steady-state path and hence considered in transitional dynamics, even in the long term; we prefer to run estimations in the long term, being suitable for the approach of endogenous theory, and we stay away from calibrated elasticities. The only calibrations adopted in some variables are based on data averages.

Bentour (2021b) reported detailed literature on the public debt threshold, which reported different numerical thresholds revealing sensitivity to the countries' sample (for panel regressions), to the period and the country on individual levels (for single-country regressions). The importance of the debt limits has already been taken into consideration before the recent financial crisis, first by the Europeans in the establishment of the European Monetary Union (Maastricht criterion of 60%), and by multilateral institutions in designing their loan programmes, especially the International Monetary Fund and the World Bank institutions. These Breton Woods Institutions designed an approach called debt sustainability analysis (DSA) for low-income countries (LIC) and market access economies (i.e. emerging economies and advanced economies). This practical framework became operational and was applied especially to LIC in 2002 (IMF and the World Bank, 2005; 2013). However, since the 2008 crisis, this approach is permanently revised and has been updated to include other highly indebted countries, which is the case for advanced countries (IMF, 2013). The DSA is a kind of stress test for debt sustainability based on limits of some important financial indicators, namely, bond yield spreads, external financing requirements (as a percentage of GDP), public debt held by non-residents (share of total), public debt in foreign currency (share of total) and annual change in the share of short-term public debt at original maturity. A safer benchmark gross government debt of 60% and 70% of GDP was calculated for the groups of emerging countries and advanced countries, respectively. The 70% limit was able to reach 85% for the group of advanced countries. This threshold is less than the 90% threshold that fits all countries suggested by Reinhart and Rogoff (2010).

In this section we compare our results with some results produced in the recent literature that have adopted modelling techniques derived from economic theory, although the approaches differ. Some papers that conclude debt limits, and then the fiscal space defined as these debt limits minus actual debt, are influential and have important policy implications (Ostry et al., 2010, 2015; Ghosh et al., 2013; Pienkowski, 2017). For many of these researchers, the debt limit is assessed around the period of the financial crisis of 2008/2009. Therefore, to compare our results with those raised by some authors, we limit this comparison to a very short period or a specific year of comparison (for example, around the 2008 financial crisis).

Returning to our earlier results, Table 6 shows the average limits over the crisis period, 2009–12, for the 20 countries in the sample. The second column shows observed average debt over the period 2009–12, the third, fifth and seventh columns show, respectively, the averages of simulated potential debt over the same period considering the economic performances proper to each country (Potential1), then simulation based on the calibration of economic growth and long-term interest rates to the Great Moderation Era (1985–2015) performances (Potential2), and the simulation calibrating long-term interest rates and economic growth to those of the United States for all countries (Potential3). The other columns, Gap1, Gap2 and Gap3, present the difference between the previous simulations (Potential1, Potential2 and Potential3) and observed debt (actual), respectively. When actual debt overcomes simulated debt, the gap is negative, and the country should curve the accumulation of debt by rapid de-leveraging.

A negative gap over the years of the financial crisis (2008–12) is recorded for Belgium, the United Kingdom and Spain, and highly negative for Japan, Greece and Italy (Gap1). Portugal, Germany and France also recorded very low but positive gaps. Considering the Great Moderation Era, Gap2 is negative for Portugal, in addition to the previous countries cited for Gap1, except the United Kingdom and Japan. The latter recorded high potential debt in contrast to high negative potential debt for Gap1. Considering the third simulation, Greece, Italy, Japan and Portugal are countries with worrying debt, even considering the United States' benchmark for economic growth and interest rate performances. However, some countries' potential debt has significantly increased in this simulation. This is the case for Australia, New Zealand, France, the United Kingdom, Spain and, to a lesser extent, Germany. Other countries, however, have seen their potential debt decrease in this last simulation, namely, Austria and Switzerland.

Table 6. Simulated potential debt following the 2009–2012 crisis

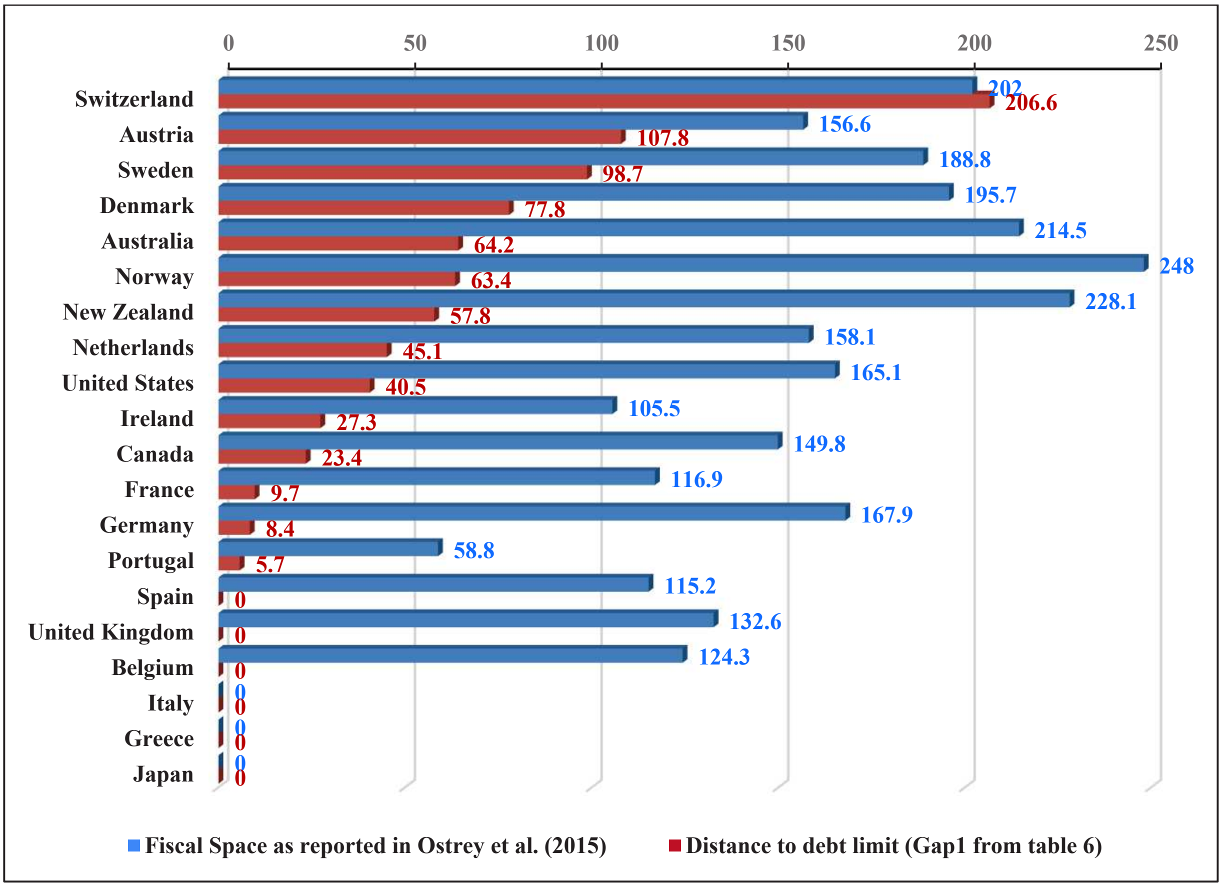
Sample: 2009-2012	Actual	Potential1	Gap1	Potential2	Gap2	Potential3	Gap3
Switzerland	48.4	255	206.6	283.8	235.4	131.8	83.4
Austria	71.9	179.8	107.8	133.2	61.3	112.1	40.2
Sweden	39.5	138.2	98.7	121.9	82.4	148.8	109.3
Denmark	41.6	119.4	77.8	100.9	59.3	126.8	85.2
Australia	20.3	84.4	64.2	95.6	75.4	126.6	106.4
Norway	43.5	107	63.4	86.1	42.6	111.7	68.2
New Zealand	30.2	87.9	57.8	91.1	60.9	153.1	123
Netherlands	62.8	107.8	45.1	126.7	64	110.8	48.1
United States	91.5	132	40.5	149.2	57.7	132	40.5
Ireland	84.5	111.8	27.3	310.1	225.6	141	56.5
Canada	80.8	104.2	23.4	110.8	30	122.5	41.7
France	81.2	90.9	9.7	103.1	21.9	124.7	43.4
Germany	76.8	85.2	8.4	102	25.2	95.8	19
Portugal	96.8	102.5	5.7	70.5	**-26.3**	86.8	**-10**
Spain	62.4	58.9	**-3.5**	59.5	**-2.9**	92.7	30.3
United Kingdom	73.2	62.7	**-10.5**	84	10.8	103.4	30.3
Belgium	96.5	76	**-20.5**	84.3	**-12.2**	98.3	1.8
Italy	117.2	60.9	**-56.3**	41.1	**-76.1**	63	**-54.2**
Greece	144.2	1.5	**-142.7**	1.1	**-143.1**	2.1	**-142**
Japan	215.5	-432.1	**-647.5**	876.5	661	185	**-30.4**

Note: Countries are filtered, according to the column Gap1, from high values to lower values.

In the same way, Ghosh et al. (2013) use the approach of Ostry et al. (2010) to calculate the debt limits for advanced countries and to produce a fiscal space defined as the difference between that debt limit and the actual debt-to-GDP ratio, based, in particular, on the interest rate and economic growth differential and considering the primary balance. They find that Greece, Italy, Japan and Portugal have no fiscal space as their debt is unsustainable. However, the other countries have enough fiscal space, especially given that the assessed debt limits range from minimum values of 152% and 154% for Canada and Germany, respectively, to high values of 246% and 263% for Ireland and Norway, respectively. For the set of countries with positive fiscal space, the latter values are around low values of 50% to 70% recorded for the United States, Ireland and Belgium, medium values from 75% to 100% recorded for the United Kingdom, France and Germany, and higher values of higher than 100%, especially in northern countries (Norway, Sweden and Denmark) and Australia and New Zealand. These results are generally in line with our results in Table (6).

The same exercise was updated by the international rating agency, Moody's, adopting the same approach as Ostry et al. (2010) and Ghosh et al. (2013), to calculate a distance to debt limit for a sample of advanced and emerging economies. The sample of countries also covers the 20 advanced countries that we studied in this paper, in addition to other countries. The results are reproduced in Ostry et al. (2015) in a figure summarizing these distances to debt limit, which are reproduced in Figure 13. The latter indicates that Japan, Italy and Greece have zero fiscal space, which was also confirmed by our results in Table 6 (Gap1). Belgium, Spain and the United Kingdom also report negative differences between potential debt and actual debt in Table (6), concordant to some extent with Moody's results, where fiscal space is very low, except for the United Kingdom. Our results (Table 6) also show that Switzerland, Austria, Sweden, Denmark, Australia, Norway, New Zealand, The Netherlands, The United States and, to some extent, Ireland and Canada have safer potential debt higher than actual debt, while France, Germany and Portugal have average potential debt approaching actual debt. These results are also generally in line with Moody's calculations, as reported in Ostry et al. (2015), which are reproduced in Figure 13.

Figure 13. Fiscal space: distance to debt limit (percentage points)



Note: For comparison purposes, distance to debt limit, as defined in Ostry et al. (2015), is reproduced and reported for our sample jointly with the distance from actual to potential debt calculated by our method (Gap1 in Table 6). For negative Gap1 (potential tolerable debt below observed debt), we report zero instead of negative numbers.

In another IMF paper (Pienkowski, 2017), the debt limit for advanced countries is found to be 137% of GDP. This debt tolerance can be enhanced by issuing GDP-linked bonds, which can increase this limit to 238% of GDP for all advanced countries (when the bonds issuance is 100% linked to GDP). The author concludes that this linked debt to GDP clearly shows that there is no one-size-fits all, which endorses our results.

Another important point revealed in this paper is the effects of the interest rates and economic growth differential (gap) in shaping the curve of potential public debt over time. The effects are higher on potential public debt in the Great Moderation Era (1986–2015) than in 1960–85. This is due to the gap being higher, on average, in the 1960–85 period and narrowing in the Great Moderation Era (Table 4). This latter period is characterized, on average, by lower interest rates, but also lower economic growth compared to 1960–85, which makes the differential between long-term interest rates and economic growth lower than the one in 1960–85, as revealed in Table 4. As a consequence of lower interest rates, the Great Moderation Era has played a bigger role in the accumulation of public debt that countries nowadays struggle to decrease (Cecchetti el al., 2011). A recent debate was triggered by Blanchard (2019) assuming that, on average (opposite to the case here for long-term interest rates), interest rates are lower along history than economic growth, which makes debt safer in advanced economies and means that there is nothing to be concerned about. Some economists interpreted this as an appeal for fiscal expansion and avoiding fiscal austerity. More details of this debate are presented in the (Bentour, 2021a), as well as a recent discussion about the type of interest rates by Blanchard (2019) in his analysis.

The third result is that potential government investment to GDP ratios are, on average, higher than the observed government investment ratios, and they tend to decrease over time. This may indicate that, first, countries are not reaching their potential productive investments, and, second, that the issued public debt may drift from financing productive capital to other government expenses as an enhancement of the welfare state needs and ageing population expenditure. In fact, the size of social spending in OECD countries increased from 18% in 1980 to 26% in 2014 (Alesina and Passalacqua, 2015).

Despite the importance of such results, some caveats should be considered, and future development could enhance the results of our approach. First, we assumed that potential debt is only issued to finance potential public capital derived by the model, while the finance of the other current government expenditure is supposedly filled by collecting taxes. This may have led to generating simulated higher potential public debt, which stays high, above actual debt, especially in good times for many countries. However, tax revenue is not sufficient to compensate the financing needs of total expenditure. For example, according to the OECD database website, tax revenue in France was around 45% of GDP in 2015 (above an average of 34% for OECD countries), while general expenditure stood at around 57% (social expenditure 43% plus social protection 14%). Government investment was around 3.5%. Hence, a deficit of approximately 11 points of GDP was not compensated by taxes (around 20% of additional taxes are needed to finance such needs). If we adjust the potential debt to consider such a fact (let us reduce potential government expenditure for all countries by an extra 20% of their actual taxes),[60] the potential public debt is significantly reduced, on average, for example, to 35% over the period 1960–2015, 72% in Germany, 21% in Italy, 50% in Ireland, 38% in Spain, 62% in Canada and 115% in the USA.

[60] The results of this exercise for all countries are reproduced in Figure B.4 in the appendix.

Without considering this fact, averages are higher and are, respectively, 131% in France and Germany, 143% in Italy, 163% in Ireland, 105% in Spain, 139% in Canada and 187% in the United States. Some other countries, such as the United Kingdom, saw potential debt significantly reduced to very low levels.

In this paper, the adopted theoretical framework of the Cobb-Douglas production function has focused on the long-term approach to simulate the potential debt that a country could target without undermining its public finances. However, this long-term approach omits the short-term effects of fiscal policies, which are highly determinant in public debt management. The framework of endogenous growth models could be suitable for long-term analysis, while debt related to fiscal-policy short-term effects is omitted. Furthermore, we derived potential debt from potential public productive investment, neglecting the effects of other public expenditure such as social spending in education, justice, health, and so on. In fact, as stated by Corsetti and Roubini (1996), *"The productive public spending can be thought of as encompassing very different types of publicly provided goods and services, such as justice, enforcement of law and contracts, police services, educational services and government research activities"*. Social spending in education, health, justice, and others, is to enhance human capital, and to create qualified political and economic institutions, which develop the business environment. The rule of law and order, as well as security, are also crucial determinants ensuring stability for the business environment. Taking this into consideration, it is highly recommended to consider the effects of such categories of expenditure and to analyse their impacts on economic growth in both the short and long terms.

5. Conclusion

Contrary to articles, which examined the purely statistical–econometric relationship between economic growth and public debt, albeit with advanced econometric tools (Bentour, 2021b), this paper has extended the investigation to the role of theory-based models, taking into consideration country-specific fundamentals. Therefore, we first surveyed the effects of public debt on economic growth in different classes of theoretical and empirical public debt models (OLG, ILA, endogenous growth models, new Keynesian DSGE models, and the normative versus the positive approach). In the second part, an endogenous model was applied to a sample of 20 developed economies.

In this model we simulated a parameterized formula for a potential/limit debt that a country could target to finance its productive investments. These limits, linked to economic growth and public capital productivity, as well as the interest rate, are dynamic, country- and time-specific and tend to evolve contrary to actual accumulated public debt. Simulated public debt, in particular, drops under actual levels of debt in times of crisis, especially for many advanced countries severely affected by the crisis. This sends a clear message of policy recommendation that countries are safe from the danger of public debt as long as potential debt (simulated/targeted) stays higher than actual public debt. The results show that many countries are under the stress of public debt, especially after the financial crisis of 2008. For countries such as Ireland, this stress is short-lived, and potential debt quickly remains higher than actual debt following the redressing in economic activity a few years after the crisis. However, for many other countries, the effects are prolonged. The results also revealed that countries such as Greece, Belgium and, to some extent, Italy, had problems of accumulated debt, in the early 1980s for the two first countries, and the early 2000s for Italy.

The potential (limit) public debt is country-specific and evolves contrary to actual accumulated debt over time. Indeed, it moves in line with the public capital productivity trend, which historically tends to decline while actual public debt is rolling over. The story of the debt limit was intensely debated at the forefront of the 2008 financial crisis. The different results found in the literature supporting the debt limit existence, whether for all countries (or at least for countries of the same level of development), or case-specific countries, tend to discuss the threshold debt generally, as a one-size-fits-all for countries, whether determined endogenously or set exogenously. However, the added value of the present work is that the debt limit is an endogenous parametrized function linked to the economic returns (productivity) of public capital financed by such issued debt. Another point to note is that, although some authors found that the endogenous debt limit, whenever it exists, is country- and time-specific, using Hansen's (2017) regression kink method as in Bentour (2021b), this paper added that potential debt limits are determined yearly based on the country's principal macroeconomic fundamentals.

References

Acemoglu D. and Robinson J. A. 2010. "The Role of Institutions in Growth and Development", Review of Economics and Institutions, vol. 1(2), pp. 1-33.

Acemoglu D., Johnson S. and Robinson J. A. 2001. "The Colonial Origins of Comparative Development: An Empirical Investigation", American Economic Review, vol. 91(5), pp. 1369-1401.

Aiyagari S. R. 1985. "Deficits, interest rates, and the tax distribution", Quarterly Review, Federal Reserve Bank of Minneapolis, vol. 9(1), pp. 5-14.

Aiyagari S. R. 1989. "Equilibrium existence in an overlapping generations model with altruistic preferences", Journal of Economic Theory, vol. 47(1), pp. 130-152.

Aiyagari S. R., Marcet A., Sargent, T. and Seppala J. 2002. "Optimal Taxation without State-Contingent Debt", Journal of Political Economy, vol. 110(6), pp. 1220-1254.

Aiyar S. and Feyrer J. 2002. "A Contribution to the Empirics of Total Factor Productivity", Dartmouth College working paper.

Alesina A. and Passalacqua A. 2015. "The Political Economy of Government Debt", NBER working paper 21821, National Bureau of Economic Research.

Alesina A. and Perotti R. 1994. "The Political Economy of Budget Deficits", NBER working paper 4637, National Bureau of Economic Research.

Alesina A. and Tabellini G. 1990. "A positive theory of fiscal deficits and government debt", Review of Economic Studies, vol. 57(3), pp. 403-414.

Aoki M. 2007. "Endogenizing institutions and institutional changes", Journal of Institutional Economics, vol. 3(1), pp. 1-31.

Arrow K. J. 1962. "The Economic Implications of Learning by Doing", Review of Economic Studies, vol. 29, pp. 155-173.

Arrow K. J. and Kurz M. A. (1970). "Public Investment, the Rate of Return, and Optimal Fiscal Policy", Baltimore, Johns Hopkins University Press.

Barro R. J. 1974. "Are Government Bonds Net Wealth?", Journal of Political Economy, vol. 82(6), pp. 1095-1117.

Barro R. J. 1976. “Perceived wealth in bonds and social security and the Ricardian equivalence theorem: Reply to Feldstein and Buchanan”, Journal of Political Economy, vol. 84(2), pp. 343-350.

Barro R. J. 1979. “On the determination of the public debt”, Journal of Political Economy, vol. 87(5), pp. 940-971.

Barro R. J. 1990. “Government spending in a simple model of endogeneous growth”, Journal of Political Economy, vol. 98(S5), pp. 103-125.

Barro R. J. 1996. “Democracy and Growth”, Journal of Economic Growth, vol. 1(1), pp. 1-27.

Barro R. J. and Lee J. W. 1993. “International comparisons of educational attainment”, Journal of Monetary Economics, vol. 32(3), pp. 363-394.

Barro R. J. and Lee J. W. 2013. “A new data set of educational attainment in the world, 1950–2010”, Journal of Development Economics, vol. 104, pp. 184-198.

Barro R. J. and Sala-i-Martin X. 1992. “Convergence”, Journal of Political Economy, vol. 100(2), pp. 223-251.

Barro R. J. and Sala-i-Martin X. 1997. “Technological Diffusion, Convergence, and Growth”, Journal of Economic Growth, vol. 2(1), pp. 1-26.

Barro R. J. and Sala-i-Martin X. 2003. Economic Growth, 2nd edition, MIT Press.

Barta Z. 2018. “In the Red: The Politics of Public Debt Accumulation in Developed Countries”, Ann Arbor: University of Michigan Press.

Benhabib J. and Spiegel M. 1994. “The roles of human capital in economic development: evidence from aggregate cross-country data”, Journal of Monetary Economics, vol. 34, pp. 143-173.

Benigno P. and Woodford M. 2003. “Optimal Monetary and Fiscal Policy: A Linear Quadratic Approach”, NBER working paper 9905, National Bureau of Economic Research.

Bentour E. M. 2021a. “The effects of public debt accumulation and business cycle on government spending multipliers”, Applied Economics, forthcoming.

Bentour E. M. 2021b. “On the public debt and growth threshold: one size does not necessarily fit all”. Applied economics, vol. 53(11), pp. 1280-1299.

Bernanke B. 2004. “The Great Moderation”, Remarks before the Meetings of the Eastern Economic Association, Washington, D.C., February 20.

Blanchard O. 1985. “Debt, Deficits, and Finite Horizons”, Journal of Political Economy, vol. 93(2), pp. 223-247.

Blanchard O. 2018. “On the future of macroeconomic models”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 43-54.

Blanchard O. 2019. “Public Debt and Low Interest Rates”, American Economic Review, vol. 109(4), pp. 1197-1229.

Bloom N., Jones C. I., Van Reenen J. and Webb M. 2019. “Are Ideas Getting Harder to Find?”, NBER working paper 23782, National Bureau of Economic Research.

Borensztein E., De Gregorio J. and Lee J-W. 1998. “How does foreign direct investment affect economic growth?”, Journal of International Economics, vol. 45(1), pp. 115-135.

Bräuninger M. 2005. “The Budget Deficit, Public Debt, and Endogenous Growth”, The Journal of Public Economic Theory, vol. 7(5), pp. 827-840.

Buiter W. H., 1988. “Death, Birth, Productivity Growth and Debt Neutrality”, The Economic Journal, vol. 98(391), pp. 279-93.

Carlin W. and Soskice D. 2018. “Stagnant productivity and low unemployment: stuck in a Keynesian equilibrium”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 169-194.

Cass D. 1965. “Optimum Growth in an Aggregate Model of Capital Accumulation”, Review of Economic Studies, vol. 32(3), pp. 233-240.

Cecchetti S. G., Mohanty M. S. and Zampolli F., 2011. “The real effects of debt”, BIS working paper 352, Bank for International Settlements.

Chapple S. 1997. “One for the X-files: A critical assessment of Professor Scully’s ‘Taxation and economic growth in New Zealand’”, Report to the Periodic Report Group, New Zealand Institute of Economic Research.

Chari V. V., McGrattan E. R. and Kehoe P. J. 2009. “The New Keynesian Models: Not Yet Useful for Policy Analysis”, American Economic Journal: Macroeconomics, vol. 1(1), pp. 242-266.

Christiano L. J., Eichenbaum M. and Evans C. L. 2005. “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”, Journal of Political Economy, vol. 113(1), pp. 1-45.

Clark T. 2009. “Is the Great Moderation Over? An Empirical Investigation”, Economic Review, Federal Reserve Bank of Kansas City, pp. 5-42.

Corsetti G. and Roubini N. 1996. “Optimal Government Spending and Taxation in Endogenous Growth Models”, NBER working paper 5851, National Bureau of Economic Research.

Cukierman A. and Meltzer A. H. 1989. “A Political Theory of Government Debt and Deficits in a Neo-Ricardian Framework”, American Economic Review, vol. 79(4), pp. 713-32.

De la Croix D. and Michel P. 2010. “A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations”, Cambridge University Press.

Diamond P. A. 1965. “National Debt in a Neoclassical Growth Model”, American Economic Review, vol. 55(5), pp. 1126-1150.

Easton B. 1999. “The whimpering of the state. Policy after MMP”, University of Auckland Press.

Feldstein M. S. 1976. “Perceived Wealth in Bonds and Social Security: A Comment”, vol. 84(2), pp. 331-36.

Fuentes R. J. and Morales M. 2011. “On the measurement of Total Factor Productivity: A Latent Variable Approach”, Macroeconomic Dynamics, vol. 15, pp. 145-159.

Futagami K., Iwaisako T. and Ohdoi R. 2008. “Debt policy rule, productive government spending, and multiple growth paths”, Macroeconomic Dynamics, vol. 12, pp. 445-462.

Futagami K., Morita Y. and Shibata A. 1993. “Dynamic analysis of an endogenous growth model with public capital”, Scandinavian Journal of Economics, vol. 95, pp. 607-625.

Galí J., López-Salido J. D. and Vallés J. 2007. “Understanding the effects of Government Spending on Consumption”, European Economic Association Journal, vol. 5(1), pp. 227-270.

Ghironi F. 2018. “Macro needs micro”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 195-218.

Ghosh S. and Mourmouras I. A. 2004a. “Endogenous growth, welfare and budgetary regimes”, Journal of Macroeconomics, vol. 26, pp. 623-635.

Ghosh S. and Mourmouras I. A. 2004b. “Debt, growth and budgetary regimes”, Bulletin of Economic Research, vol. 56, pp. 241-250.

Ghosh A., Kim J., Mendoza E., Ostry J. and Qureshi M. 2013. “Fiscal Fatigue, Fiscal Space, and Debt Sustainability in Advanced Economies”, The Economic Journal, vol. 123(566), pp. F4-F30.

Greiner A. 2007. “An endogenous growth model with public capital and sustainable government debt”, Japanese Economic Review, vol. 58(3), pp. 345-361.

Greiner A. 2011. “Economic Growth, Public Debt and Welfare: Comparing Three Budgetary Rules”, German Economic Review, vol. 12(2), pp. 205-222.

Greiner A. 2012. “Public debt in a basic endogenous growth model”, Economic Modelling, vol. 29(4), pp. 1344-1348.

Greiner A. 2014. “Fiscal and Monetary Policy in a Basic Endogenous Growth Model”, Computational Economics, vol. 45(2), pp. 285-301.

Greiner A. 2015. “Public Debt, Productive Public Spending and Endogenous Growth”, Japanese Economic Review, vol. 66(4), pp. 520-535.

Greiner A. 2016. “Human capital formation and public debt: growth and welfare effects of three different deficit policies”, Journal of Economic Policy Reform, vol. 19(4), pp. 369-385.

Greiner A. and Semmler W. 1999. “An endogenous growth model with public capital and government borrowing”, Annals of Operations Research, vol. 88, pp. 65-79.

Greiner A. and Semmler W. 2000. “Endogenous growth, government debt and budgetary regimes”, Journal of Macroeconomics, vol. 22(3), pp. 363-384.

Grilli V., Masciandaro D., Tabellini G., Malinvaud E. and Pagano M. 1991. “Political and Monetary Institutions and Public Financial Policies in the Industrial Countries”, Economic Policy, vol. 6(13), pp. 341-392.

Grossman G. M. and Helpman E. 1991. “Quality Ladders in the Theory of Growth”, The Review of Economic Studies, vol. 58(1), pp. 43-61.

Haldane A. G. and Turrell A. E. 2018. “An interdisciplinary model for macroeconomics”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 219-251.

Hansen B. E. 2017. “Regression Kink with an Unknown Threshold”, Journal of Business and Economic Statistics, vol. 35 (2), pp. 228-240.

Haslag J. H. and Young E. R. 1998. “Money creation, reserve requirements, and seigniorage”, Review of Economic Dynamics, vol. 1(3), pp. 677-698.

Hendry D. F. and Muellbauer J. N. 2018. “The future of macroeconomics: macro theory and models at the Bank of England”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 287-328.

Hill R. 2008. “Optimal taxation and economic growth: a comment”, Public Choice, vol. 134, pp. 419-427.

Hodrick R. J. and Prescott E. C. 1997. “Postwar U.S. Business Cycles: An Empirical Investigation”, Journal of Money, Credit and Banking, vol. 29(1), pp. 1-16.

Hsieh C. T. and Klenow P. J. 2010. “Development Accounting”, American Economic Journal: Macroeconomics, vol. 2(1), pp. 207-223.

Hulten C. 2001. “Total Factor Productivity: A Short Biography,” in New Developments in Productivity Analysis, Charles R. Hulten, Edwin R. Dean, and Michael J. Harper, eds., Studies in Income and Wealth, vol. 63, The University of Chicago Press for the National Bureau of Economic Research, Chicago, pp. 1-47.

IMF. 2013. “Staff Guidance Note for Public Debt Sustainability Analysis in Market-Access Countries”, International Monetary Fund.

IMF. 2015. “Making Public Investment more Efficient”, Policy Report, International Monetary Fund.

IMF and The World Bank. 2005. “Operational Framework for Debt-Sustainability Assessments in Low-Income Countries—Further Considerations”.

IMF and The World Bank. 2013. “Staff Guidance Note on the Application of the Joint Bank-Fund Debt Sustainability Framework for Low Income Countries”.

Jones C. I. 1995. “R&D-Based Models of Economic Growth”, Journal of Political Economy, vol. 103(4), pp. 759-784.

Jones C. I. 2003. “Population and Ideas: A Theory of Endogenous Growth”. In P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps, Princeton University Press.

Jones C. I. 2005. “Growth and Ideas”. In P. Aghion and Durlauf S., eds, Handbook of Economic Growth, edition 1, vol. 1, chapter 16, pp. 1063-1111.

Jones C. I. 2019. “Paul Romer: Ideas, Nonrivalry, and Endogenous Growth”, Scandinavian Journal of Economics, vol. 121(3), pp. 859-883.

Jones C. I. and Romer P. M. 2010. “The New Kaldor Facts: Ideas, Institutions, Population, and Human Capital”, American Economic Journal: Macroeconomics, vol. 2(1), pp. 224-245.

Kennedy P. E. 2000. “On measuring the growth-maximizing tax rate”, Pacific Economic Review, vol. 5(1), pp. 89-91.

Kmenta J. 1967. “On Estimation of the CES Production Function”, International Economic Review, vol. 8, pp. 180-189.

Krugman P. 2018. “Good enough for government work? Macroeconomics since the crisis”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 156-168.

Kydland F. E. and Prescott E. C. 1977. “Rules Rather than Discretion: The Inconsistency of Optimal Plans”, Journal of Political Economy, vol. 85(3), pp. 473-492.

Leith C. and von Thadden L. 2008. “Monetary and fiscal policy interactions in a New Keynesian model with capital accumulation and non-Ricardian consumers”, Journal of Economic Theory, vol. 140(1), pp. 279-313.

Leith C. and Wren-Lewis S. 2013. “Fiscal sustainability in a new Keynesian model”, Journal of Money, Credit and Banking, vol. 45(8), pp. 1477-1516.

Lindé J. 2018. “DSGE models: still useful in policy analysis?”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 269-286.

Lucas R. E. 1976. “Econometric policy evaluation: A critique”, Carnegie Rochester Conference Series on Public Policy, vol. 1(1), pp 19-46.

Lucas R. E. 1988. “On the Mechanics of Economic Development”, Journal of Monetary Economics, vol. 22, pp. 3-42.

Lucas R. E. and Stokey N. L. 1983. “Optimal Fiscal and Monetary Policy in an Economy Without Capital”, Journal of Monetary Economics, vol. 12, pp. 55-93.

Maebayashi N., Hori T. and Futagami K. 2017. “Dynamic analysis of reductions in public debt in an endogenous growth model with public capital”, Macroeconomic Dynamics, vol. 21, pp. 1454-1483.

Mankiw N. G. 2000. “The Savers-Spenders Theory of Fiscal Policy”, American Economic Review, vol. 90(2), pp. 120-125.

Mayer E., Rüth S. and Scharler J. 2013. “Government debt, inflation dynamics and the transmission of fiscal policy shocks”, Economic Modelling, vol. 33, pp. 762-771.

McKibbin W. J. and Stoeckel A. 2018. “Modelling a complex world: improving macro-models”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 329-347.

Miller P. J. and Sargent T. J. 1984. “A reply to Darby”, Federal Reserve Bank of Minneapolis Quarterly Review, vol. 8, pp. 21-26.

Minea A. and Villieu P. 2013. “Debt policy rule, productive government spending, and multiple growth paths: A note”, Macroeconomic Dynamics, vol. 17, pp. 947-954.

Modigliani F. 1961. "Long-run implications of alternative fiscal policies and the burden of the national debt", Economic Journal, vol. 71(284), pp. 730-755.

Nelson R. and Phelps E. 1966. "Investment in Humans, Technological Diffusion, and Economic Growth", American Economic Review, vol. 56(1/2), pp. 69-75.

Ni S. 1999. "National Debt, Savings, and Real Interest Rates in a Neoclassical Model with Endogenous Labour Supply and Knowledge-Based Growth", The Canadian Journal of Economics, vol. 32(5), pp. 1227-1244.

North D. C. 1990. Institutional Change, and Economic Performance, Cambridge University Press.

Obstfeld M. 1997. "Dynamic Seigniorage Theory", Macroeconomic Dynamics, vol. 1(3), pp. 588-614.

Ostry J. D., Ghosh A.R. and Espinoza R. 2015. "When Should Public Debt Be Reduced?", IMF Staff Discussion Note SDN/15/10, International Monetary Fund.

Ostry J. D., Ghosh A. R., Kim J. I. and Qureshi M.S. 2010. "Fiscal Space", IMF Staff Position Note 10/11, International Monetary Fund.

Persson T. and Svensson L. E. O. 1989. "Why a Stubborn Conservative would Run a Deficit: Policy with Time-Inconsistent Preferences", Quarterly Journal of Economics, vol. 104(2), pp. 325-345.

Phelps E. S. 1961. "The Golden Rule of Accumulation: A Fable for Growthmen", American Economic Review, vol. 51, pp. 638-643.

Pienkowski A. 2017. "Debt Limits and the Structure of Public Debt", IMF working paper, WP/17/117, International Monetary Fund.

Ramsey F. P. 1928. "A Mathematical Theory of Saving", The Economic Journal, vol. 38(152), pp. 543-559.

Razzak W. A. and Bentour E. M. 2013. "Do developing countries benefit from foreign direct investments? An analysis of some Arab and Asian countries", Review of Middle East Economics and Finance, vol. 9(3), pp. 357-388.

Reis R. 2018. "Is something really wrong with macroeconomics?", Oxford Review of Economic Policy, vol. 34(1-2), pp. 132-155.

Rodrik D., Subramanian A. and Trebbi F. 2004. "Institutions Rule: The Primacy of Institutions over Geography and Integration in Economic Development", Journal of Economic Growth, vol. 9(2), p.131-165.

Romer, P. M. 1986. “Increasing Returns and Long-Run Growth”, Journal of Political Economy, vol. 94(5), pp. 1002-1037.

Romer, P. M. 1990. “Human capital and growth: Theory and evidence”, Carnegie-Rochester Conference Series on Public Policy, Elsevier, vol. 32(1), pages 251-286.

Rossi R. 2014. “Designing monetary and fiscal policy rules in a new Keynesian model with rule-if-thumb consumers”, Macroeconomic Dynamics, vol. 18(2), pp. 395-417.

Roubini N. and Sachs J. D. 1989. “Political and economic determinants of budget deficits in the industrial democracies”, European Economic Review, vol. 33(5), pp. 903-938.

Rupert P. and Šustek R. 2019. “On the mechanics of New-Keynesian models”, Journal of Monetary Economics, vol. 102, pp. 53-69.

Samuelson P. 1958. “An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money”, Journal of Political Economy, vol. 66(6), pp. 467-482.

Schmitt-Grohe S. and Uribe M. 2004. “Optimal Monetary and Fiscal Policy under Sticky Prices”, Journal of Economic Theory, vol. 114, pp. 198-230.

Scully G. W. 1996. “Taxation and economic growth in New Zealand”, Pacific Economic Review, vol. 1(2), pp. 169-177.

Scully G. W. 1999. “Unfinished reform: taxation and economic growth in New Zealand”, Journal of Private Enterprise, vol. 14(2), pp. 92-114.

Scully G. W. 2003. “Optimal taxation, economic growth, and income inequality”, Public Choice, vol. 115, pp. 299-312.

Sims C. A. 1972. “Money, Income, and Causality”, American Economic Review, vol. 62(4), pp. 540-552.

Smets F. and Wouters R. 2007. “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach”, American Economic Review, vol. 97(3), pp. 586-606.

Solow R. 1956. “A Contribution to the Theory of Economic Growth”, Quarterly Journal of Economics, vol. 70, pp. 65-94.

Stiglitz J. E. 2018. “Where modern macroeconomics went wrong”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 70-106.

Stock J. H. and Watson M. W. 2003. “Forecasting Output and Inflation: The Role of Asset Prices”, Journal of Economic Literature, vol. 41(3), pp. 788-829.

Swan T. W. 1956. “Economic growth and capital accumulation”, Economic Record, vol. 32(2), pp. 334-361.

Terra C. 2015. “Principles of International Finance and Open Economy Macroeconomics: Theories, Applications and Policies”, Academic Press, 1st edition.

Turnovsky S. J. 1997. “Fiscal policy in a growing economy with public capital”, Macroeconomic Dynamics, vol. 1, pp. 615-639.

Turnovsky S. J. and Brock W. A. 1980. “Time consistency and optimal government policies in perfect foresight equilibrium”, Journal of Public Economics, vol. 13(2), pp. 183-212.

Vines D. and Wills S. 2018a. “The rebuilding macroeconomic theory project: an analytical assessment”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 1-42.

Vines D. and Wills S. 2018b. “The financial system and the natural real interest rate: towards a ‘new benchmark theory model’”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 252-268.

Weil P. 1989. “Overlapping families of infinitely-lived agents”, Journal of Public Economics, vol. 38(2), pp. 183-198.

Weil P. 2008. “Overlapping Generations: The First Jubilee”, Journal of Economic Perspectives, vol. 22(4), pp.115-134.

Wren-Lewis S. 2018. “Ending the microfoundations hegemony”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 55-69.

Wright R. 2018. “On the future of macroeconomics: a New Monetarist perspective”, Oxford Review of Economic Policy, vol. 34(1-2), pp. 107-131.

Yaari M. 1965. “Uncertain lifetime, life insurance, and the theory of the consumer”, Review of Economic Studies, vol. 32, pp. 137-150.

Yakita A. 2008. “Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting”, Journal of Public Economics, vol. 92(2008), pp. 897-914.

Appendix A

Table A.1. Summary and features of the models of public debt and fiscal policy

Authors	Type	Main Features	Main hypotheses	Results
Diamond (1965)	OLG	- Two periods of infinite long-life agents living for 2: working in the 1st and retiring in the 2nd. - Study the impact of domestic/foreign debt on the interest rate and utility level.	- Governments have a one period maturity. - Debt pays the current interest rate. - Taxes are lump-sum. - Debt labor ratio is kept constant.	- Domestic debt raises interest rate and lowers utility in the efficient case. - Results for the inefficient case depend on the existence of the foreign debt with domestic debt. - Foreign debt widen the gap between the interest rate and growth, lowers utility in the efficient case. - Results for utility are mixed for the inefficient case. Debt Swap has the same effects.
Barro (1974)	OLG	- The model's features are mainly based on the Samuelson-Diamond properties. - Study the effects of public debt on interest rate and utility in steady state equilibrium.	- The model's hypotheses are mainly based on the Samuelson (1958) and Diamond (1965) models' assumptions.	- Government bonds effects on the interest rate and utility does not necessarily hold and depends especially on the existence of operative altruistic links (bequest motives) between generations. - Intergenerational transfers guarantee debt-neutrality as the debt-for-tax swap does not affect the resource allocations and interest rates of the current generation.
Blanchard (1985)	ILA	- A finite lived agents' horizon. - Studies the effects of debt and deficit accumulation. - Design an "index of fiscal policy" capturing the effects of current and expected fiscal policy.	- Taxes are lump-sum. - Constant probability of death.	- Debt increase (in the steady state): changes the foreign assets in agents' wealth in an open economy, and, decreases the level of capital and consumption in a closed economy. - Taxes decrease: raises wealth and consumption. Effect is larger the longer taxes are shifted to future generations. Debt and taxes increases create an initial wealth effects on consumption, leading to capital decumulation which makes capital and consumption lower in the new steady-state level.
Aiyagari (1985)	OLG	- Use a modified version of the OLG model of Samuelsson (1958). - Assess the Ricardian equivalence validation.	- Enabling changeable interest rate to deficit policy regimes. - Taxes distributed differently across heterogeneous agents.	- Ricardian equivalence validation depends on the way taxes are distributed among agents and not on the length of the lifetime horizon. The debt-neutrality holds if the tax distribution does not change agents' wealth allocations. A higher level of spending can be financed by debt at an unchanged interest rate, if taxes' distribution maintains wealth distributions.
Buiter (1988)	OLG	- The model is based on the Yaari-Blanchard models. - Evaluates the effects of government bonds on economic agents' wealth.	- Private consumption behavior is modelled following Yaari (1965) and Blanchard (1985) approaches.	- The equilibrium is independent of the pattern of the public debt and lump-sum taxation over time. - The difference between expected government tax base and the future tax base of the current individuals explains the lump-sum taxation variations over time. Agents not linked to the future generations by bequest transfers omit their successors' resources from their budget constraint. - Debt-neutrality holds if and only if the population growth and the probability of death sums to zero. Under this condition, a non-zero labor productivity rate will not destroy this debt-neutrality.

Weil (1989)	ILA	- A model of "overlapping families of infinitely lived agents", aims to show that: 1st, the "infinite lifetime horizon" assumption do not necessarily imply debt-neutrality; 2nd, the "finite horizons" don't necessarily violate debt-neutrality.	- Assumes new cohorts over time, and new families aren't linked to the existing by transfers. - Operative transfers between some but not all agents result in partial linkages leading to infinite economic horizon. - Agents' infinite lifetime is assured by the arrival of the families who are not linked by transfers.	- The equilibrium interest rate hinges on the government financing decision (government bonds by levying lump-sum taxes in this case) and speed of new arrival cohorts/families. - Government bonds increase the equilibrium interest rate for a positive non-zero population growth rate: expected taxes to pay the issued debt are compensated by generations that are not yet born and are not considered by the current generations. The real interest rate must hence rise to maintain aggregate consumption at its market-clearing level. Infinite lifetimes are so not inconsistent with the violation of the debt-neutrality assumption. Also, finite horizons do not necessarily violate the latter.
Barro (1990)	EGT	- Aims at establishing the effect of public spending and government debt policies on utility and long-term growth in a perspective of endogenous growth theory. - The model studies a closed economy with infinite lifetime agents and intertemporal preferences modelled by a utility function.	- Adopts an AK modeling framework assuming constant returns to scale and including the flows of public services as input in the production function. - Argues that private inputs are not a close substitute for public inputs as some public activities are difficult to be insured by private firms (defense, law and order), or the service is non-rival (ideas), or as external effects cause lower private output (basic education). - Assumes timely balanced-budget.	- In case of a Cobb-Douglas production function, $\frac{y}{k} = A\left(\frac{g}{k}\right)^{\alpha}$: an increase in taxes reduces the long-term growth while an increase in expenditures raises it by raising marginal productivity of capital. - The two effects cancel for the optimal government size equalizing government expenditure to tax rate and the share of the capital $\left(\frac{g}{y} = \tau = \alpha\right)$. The growth rate function of the government size is an inverted U curve: for a government small size, raising expenditures' effect dominates raising tax rate's effect, while for a large government size, the negative effect of taxes on growth dominates. - Maximum utility corresponds to same conditions maximizing growth if the elasticity of substitution of y to g = 1. - In non-Cobb-Douglas case: maximum growth depends on the elasticity of substitution between public services and private capital and, the relative size maximizing utility exceeds the one maximizing growth rate if and only if the magnitude this elasticity substitution is superior to 1.
Futagami et al. (1993)	EGT	- Same approach as in Barro (1990). Study the effect of public capital stock (instead of government services flows) and public debt policies on utility and growth.	- Adopts an AK modeling framework. - The model includes the stock of public capital as an input in the production function. - Assumes timely balanced-budget.	- Public investment stimulates aggregate production indirectly via the accumulated stock instead of flows as does Barro (1990). The introduction of the productive public stock allows dynamic transitional effects analysis instead of being restricted to the steady state analysis as in Barro (1990). - Tax rate maximizing economic growth rate is higher than the one maximizing utility.
Corsetti and Roubini (1996)	EGT	- Multi-sector model encompassing productive public spending to assess optimal fiscal policy in the approach of endogenous growth models. - Incorporate a separate human capital accumulation as a second sector. - The production function is a Cobb-Douglas with physical capital, human capital and flows of public goods.	- Assume unbalanced budget constraint allowing government borrowing/lending. - 4 models are derived depending whether: public spending as input in the output sector, affects the productivity of capital (model 1); affects the productivity of human capital (model 2); or public spending as input in the human capital sector, affects respectively the same previous variables (models 3 and 4).	- Distinguish an optimal tax rates on both types of capital under different assumptions on technology and distribution and analyze the welfare properties of public debt and assets. - If the government spending is an input in the production function only (not in the human sector), the optimal choice of the government spending leads to a constant ratio of expenditures to output in every time independently of the tax policy. This ratio is less than the one found by Barro (1990) $\left(\frac{g}{y} = \alpha(1-\varepsilon) < \alpha \ as \ \varepsilon > 0\right)$. In model 1, the optimal tax on human capital is zero and the one on physical capital is positive. This result is inverted in model 2. In both models (1 and 2), a government optimal choice of both government spending and tax rates leads to an instantaneous balanced budget and the optimal public debt is zero.

Benigno and Woodford (2003); Schmitt-Grohe and Uribe (2004)	NKT	- New Keynesian models augmented by the government's budget constraint. - Aim at analyzing the optimal response of government debt to shocks focusing on the type of the involved time-inconsistency policy.	- New Keynesian framework based on Christiano et al. (2005) and Smets and Wouters (2007). - Assume sticky prices in the short run. - Expenditures are financed by ***non-distortionary*** taxes and or by issuing only nominal risk-free debt.	- Optimal public debt would follow a random walk path if the government can achieve a time-inconsistent policy commitment. Under the assumption of price stickiness, the government (social planner) chooses to rely more heavily on changes in income tax rates rather than using surprises as a shock absorber of unexpected innovations in the fiscal budget. - The distortions introduced by tax changes are diminished by spreading them over time which induces a near random walk property in tax rates and public debt.
Leith and Wren-Lewis (2013)	NKT	- New Keynesian models augmented by the government's budget constraint. - Aim at analyzing the optimal response of government debt to shocks involving time-inconsistency policy and its implications for discretionary policies.	- New Keynesian (DSGE) framework based on Christiano et al. (2005) and Smets and Wouters (2007). - The model assumes sticky prices in the short run. - Public spending is financed by ***distortionary taxes*** and/or by debt.	- Optimal pre-commitment policy allows debt to follow a random walk path in the steady state. - However, under sticky prices framework, government is tempted to use its policy instruments to change the steady state level debt in the initial period. Debt is curved to initial efficient steady state to encounter this temptation and thus deter the public debt to follow random walk path if following shocks; the new steady state debt equates the original (efficient) debt level despite there is no explicit debt target in the government's objective function. Debt stabilization instruments depend on the degree of nominal inertia. The size of the debt stock and welfare consequences of introducing debt are negligible for pre-commitment policies but can be significant for discretionary policies.
Mayer et al. (2013)	NKT	- New Keynesian model analyzing the responses of macroeconomic variables, in the steady state, to a government spending shock in the presence of positive levels of government debt.	- Derived assumptions of New Keynesian models. - The model assumes a fraction of the household sector to follow a rule-of-thumb behavior as in Gali et al. (2007) (non-Ricardian agents).	- Large government debt in steady state impact the sign and size of short/medium run fiscal multipliers, depending on the horizon's evaluation of these multipliers. - Presence of dynamic interactions between inflation and debt level in real terms (absent in standard New Keynesian models where debt is set to zero in the steady state). - In the case of permanent debt, the fiscal policy effect becomes difficult to predict over time.
Persson and Svensson (1989)	PAD	- A principal-agent problem, aiming at comparing the policy of a conservative government, certain of been succeeded by a liberal government, to the policy where he is certain of his reelection. The level of public debt is an instrument of the current government to control the rival future government.	- Two-period perfect-foresight framework, - Assume the ruling government is certain that he will be succeeded by a liberal government. Assume homogeneity of governments' preferences towards all public goods but different preferences for different levels of the same public good.	- A conservative government may borrow more if he knew that he will be succeeded by a liberal government than it would do once certain of remaining in power in the future; A conservative government will collect less tax and leave more public debt than what the successor would prefer. - This raises the ruler government consumption than it would be if he will stay in the government, while the successor with high debt and constrained resources reduces consumption than it would be if he runs solely. Thus, the time-consistent level of government consumption is somewhere in between the two outcomes of what each of the two governments would prefer if ruling on his own.

Alesina and Tabellini (1990)	PAD	A political/positive theory of debt aiming at understanding the debt build-up and deficits in several industrialized economies Explicitly, they compare the outcome of debt accumulation and deficit in situation where governments alternate versus an outcome resulting from a social planner government supposed elected forever.	- Assume uncertainty about the nature and spending behavior of succeeding governments. - Assume different preferences for different items of public spending and a constant population of identical individuals with the same time horizon. - Individuals differ only by their preferences for public goods, supplied by the government and financed by means of distortionary taxes on labor. -Government is elected among two parties maximizing each the electorate utility function.	- Differences in political institutions, leading to different debt policies, help explain debt paths over time and across countries. Debt accumulation and deficit are emphasized by the alternation of the elected governments. Disagreement of different governments on the structure of the spending results in a deficit bias and an accumulation of debt higher than it would be in case of a social planner. - The debt left to the last period is larger in a democracy than with a social planner. - The electoral uncertainty creates a sub-optimal deficit bias. This bias is stronger for the party with the smaller probability of reappointment". The equilibrium level of government debt is higher; 1/ the higher the degree of polarization between alternating governments and 2/ the more likely the current government will not be reelected. Moreover, as the ruling government is unable to curve the taxation and expenditure policies of its successors, the law of motion of the public debt is the only way by which the fiscal policy of the ruling government can impact the policies of its successors.
Greiner (2007; 2012; 2015; 2016)	EGT	- 2007: Endogenous growth model with public capital and sustainable debt. - 2012: Endogenous growth with elastic labor supply and a government sector. - 2015: Endogenous growth model with productive public spending. - 2016: An endogenous growth model with public educational spending	- 2007: Assume the ratio of the primary surplus to gross domestic income to be a positive linear function of the debt ratio. - 2012: Government levies distortionary income tax and issues bonds to finance lump-sum transfers and non-distortionary public spending. - 2015: Rational identical households with perfect foresight maximizing their utility. - 2016: Same assumptions as in Greiner (2015).	- 2007: The model is used to derive necessary conditions for the existence of a sustainable balanced growth path and analyze growth effects of deficit financed increases in public investment. - 2012: The long-run growth rate is smaller the higher the debt ratio whenever public spending is adjusted to fulfill the government inter-temporal budget constraint. - 2015: Higher debt goes along with smaller long-run growth. Moreover, discretionary policy violates the intertemporal government budget constraint along a balanced path. - 2016: Balanced budget policy and a slight deficit policy yield higher growth than a policy where debt and GDP grow at the same rate. For high debt and low elasticity of substitution, a high deficit policy yields lower welfare than a balanced budget and a slight deficit policy.
Maebayashi et al. (2017)	EGT	- Consider a stock of capital investment in endogenous framework. - Examine how reducing public debts in the economy with large public debts affects transition of the economy and welfare.	- Builds on Futagami et al. (2008) assumptions but instead consider public capital not flows as input. - Assume the depreciation rate of public capital is zero.	- Derive an optimal target debt ratio that depends on the tax rates on wage income and consumption, and the ratio of public investment to total spending. Fiscal consolidation based on a debt ratio target rule improves welfare and this improvement is more the fastest the pace of debt reduction is. - Fiscal consolidation based on expenditure cuts jointly with a tax increase does not always improve welfare. In this case, the welfare gains (if any) are lower than those under expenditure cuts only.
Mankiw (2000)	SST	- A critic of ILA and OLG models and alternative theory mentioned as "Savers-Spenders Theory of fiscal policy" to address the neoclassical shortcomings considering that "households smooth their consumption over time" is inaccurate as many consumers are far from following complete rational expectations.	- Assume the role-of-thumb behavior (non-Ricardian) in the consumers spending. - Some individuals may enjoy long lifetime horizons (due to bequest motives) while others having short time horizons fail to smooth their consumption and accumulate wealth.	- Even if debt does not affect the steady state capital stock and income, it disrupts the income distribution and consumption leading to raise inequality between spenders and savers. A higher level of debt yields higher taxation to compensate for the interest payments on the debt. But taxes are on both savers and spenders while interest payments on debt fall on the savers side. Thus, a higher level of debt increases the steady sate income and consumption for savers and lowers it for the spenders which raise inequality between the two groups. - Many agents have no saving (data fact), so unable to smooth intertemporal consumption as reported by the ILA or OLG models. A better model would allow for such heterogeneous behaviors.
Chari et al. (2009)	CNK	- A critic to the New Keynesian models based on the study of Smets and Wooters (2007) core model.	- Critics based on the examination of Smets and Wooters (2007) properties and assumptions.	- New Keynesian models are not yet useful for public policy analysis. These models include many free parameters yielding to shocks that are dubiously structural as well as many features that are not consistent with microeconomic evidence.

Table A.2. description of variables and data sources

The IMF dataset on investment and capital stock, July 2017.	
Public Investment	General government investment (gross fixed capital formation), in billions of constant 2011 international dollars.
Public Capital	Government capital stock (constructed based on government investment flows), in billions of constant 2011 international dollars.
Private Investment	Private investment (gross fixed capital formation), in billions of constant 2011 international dollars.
Private Capital	Private capital stock (constructed based on private investment flows), in billions of constant 2011 international dollars.
GDP	Gross domestic product, in billions of constant 2011 international dollars.
Background material on data construction	The accompanying 2017 Update of the Manual "*Estimating Public, Private, and PPP Capital Stocks*" (http://www.imf.org/external/np/fad/publicinvestment/data/info122216.pdf) and (http://www.imf.org/external/pp/longres.aspx?id=4959) describes in great detail the series' definitions, the investment series' data sources, as well as the methodology in constructing the stock series. The methodology follows the standard perpetual inventory equation and largely builds on Gupta et al. (2014) "*Efficiency-Adjusted Public Capital and Growth*" and Kamps (2006) "*New Estimates of Government Net Capital Stocks for 22 OECD Countries, 1960–2001*".
Information sources	Information on public and private investment and GDP comes from three main sources: the OECD Analytical Database (August 2016 version) for OECD countries, and a combination of the National Accounts of the Penn World Tables (PWT, version 9.0) and the IMF World Economic Outlook (WEO, April 2016) for non-OECD countries. Information on country income groupings used in depreciation rates' assumptions is from the World Bank World Development Indicators.
Additional notes	Note that all data series (public investment and capital stock, private investment and capital stock, GDP, etc.) are expressed in billions of constant 2011 international dollars (purchasing power parity adjusted), using the corresponding component-specific deflators from OECD, PWT, and WB databases mentioned above.
Source of the above part of this Table	"IMF Investment and Capital Stock Dataset, 2017", drawn from http://www.imf.org/external/pp/longres.aspx?id=4959.
Other data from other sources	
Interest rates	We consider long term interest rates from OECD database statistics
Inflation	GDP deflator percent change, GDP deflators are from OECD database statistics
Tax revenues	Tax revenues as percent of GDP are from
Government expenditures	General government expenditures as percent of GDP are from OECD database website
Social expenditure	Social spending as percent of GDP are from OECD database website
Labor	We consider a working age population [15, 64] as a proxy for labor data from the World Development Indicators database of the World Bank.
Human capital	Measured by average years of schooling of population [15, 64] from Barro-Lee dataset (www.barrolee.com)

Table A.3. Descriptive statistics

Public investment (Public Gross Capital Formation) as percent of GDP																				
Sample: 1960 2015	AUS	AUT	BEL	CAN	CHE	DNK	FRA	GBR	GER	GRC	IRL	ITA	JPN	NLD	NOR	NZL	PRT	SPA	SWE	USA
Mean	2.3	4.7	3.4	3.6	3.7	6.3	4.8	3.6	3.5	3.2	4.5	3.6	8.3	4.7	3.6	4.5	4.3	3.9	5.0	4.5
Median	2.2	4.4	2.9	3.5	3.7	4.6	4.5	3.2	3.1	2.8	4.6	3.5	8.8	4.1	3.5	4.6	4.4	3.9	4.7	4.2
Maximum	3.5	6.9	6.4	4.9	4.6	14.0	6.6	6.6	6.1	5.5	6.7	5.0	11.9	8.0	4.8	6.7	6.1	5.7	7.6	7.1
Minimum	1.7	2.5	2.4	2.8	2.2	3.3	3.7	1.6	1.9	1.9	2.1	2.5	4.2	3.3	2.6	2.4	2.4	2.5	3.9	3.4
Std. Dev.	0.4	1.5	1.1	0.6	0.5	3.4	0.7	1.4	1.4	1.1	1.2	0.6	2.3	1.4	0.5	0.9	0.9	0.7	1.1	1.0
Skewness	1.0	0.1	1.3	0.4	-0.6	0.9	0.9	0.7	0.4	0.8	-0.1	0.5	-0.5	1.0	0.4	-0.5	-0.3	0.3	1.0	1.4
Kurtosis	3.7	1.5	3.4	1.9	3.7	2.2	2.9	2.1	1.7	2.3	2.0	2.8	2.2	2.6	3.1	3.0	2.5	3.2	3.0	3.8
Jarque-Bera	10.7	5.5	15.6	4.5	4.0	8.7	7.6	5.9	5.8	6.6	2.5	2.8	3.7	10.3	1.6	2.1	1.3	1.1	8.7	19.9
Probability	0.0	0.1	0.0	0.1	0.1	0.0	0.0	0.1	0.1	0.0	0.3	0.3	0.2	0.0	0.5	0.4	0.5	0.6	0.0	0.0
Private investment (Private Gross Capital Formation) as percent of GDP																				
	AUS	AUT	BEL	CAN	CHE	DNK	FRA	GBR	GER	GRC	IRL	ITA	JPN	NLD	NOR	NZL	PRT	SPA	SWE	USA
Mean	14.9	22.2	23.7	16.3	21.9	16.8	18.9	16.4	17.8	13.2	25.1	22.5	17.9	18.1	19.3	11.7	21.0	20.1	17.5	14.3
Median	14.3	22.4	23.9	16.0	22.0	17.1	18.9	16.4	17.4	13.4	24.5	21.7	18.0	18.0	19.6	11.3	21.0	19.5	17.5	13.8
Maximum	22.2	24.3	30.0	20.8	24.2	23.9	21.9	21.4	21.5	19.0	34.2	31.0	22.9	22.1	27.0	15.2	27.6	28.4	21.0	17.9
Minimum	11.1	19.1	16.2	12.7	17.2	10.7	16.9	13.1	16.0	6.7	16.5	17.3	12.4	15.3	12.8	8.8	14.9	14.9	13.3	11.1
Std. Dev.	3.0	1.5	3.1	2.1	1.6	3.5	1.2	1.9	1.5	2.6	3.9	3.4	2.3	1.6	3.5	1.8	3.0	3.5	1.7	1.9
Skewness	0.9	-0.4	-0.5	0.5	-0.9	0.2	0.4	0.4	0.9	-0.4	0.3	0.9	0.1	0.4	0.0	0.4	0.0	0.8	-0.3	0.4
Kurtosis	2.7	2.0	3.3	2.4	3.7	1.9	2.8	2.9	2.7	3.1	2.7	3.0	3.0	2.7	2.3	2.2	2.8	3.0	2.8	2.1
Jarque-Bera	7.5	3.8	2.4	2.9	9.0	3.0	1.3	1.7	6.9	1.5	0.9	7.8	0.0	1.5	1.2	3.2	0.2	5.2	1.0	3.5
Probability	0.0	0.2	0.3	0.2	0.0	0.2	0.5	0.4	0.0	0.5	0.6	0.0	1.0	0.5	0.6	0.2	0.9	0.1	0.6	0.2
Government capital stock in billions of 2011 constant US Dollars (ppp)																				
KG	AUS	AUT	BEL	CAN	CHE	DNK	FRA	GBR	GER	GRC	IRL	ITA	JPN	NLD	NOR	NZL	PRT	SPA	SWE	USA
Mean	163	188	157	464	174	221	1237	980	1490	100	54	904	3622	376	94	62	118	525	221	6752
Median	147	209	192	435	178	228	1233	1025	1671	80	50	907	3779	382	85	66	102	460	232	6229
Maximum	364	237	235	870	275	261	1921	1254	1777	216	124	1386	5991	553	198	105	219	1056	314	10889
Minimum	54	88	44	184	49	132	463	536	726	29	13	376	716	178	26	28	40	168	91	3276
Std. Dev.	82	48	66	189	74	34	459	182	333	61	34	319	1904	103	51	20	59	281	66	2257
Skewness	0.76	-0.77	-0.55	0.46	-0.25	-1.17	-0.12	-1.04	-1.11	0.74	0.81	-0.05	-0.19	-0.16	0.39	0.16	0.42	0.53	-0.51	0.37
Kurtosis	2.82	2.11	1.68	2.31	1.73	3.50	1.75	3.23	2.76	2.18	2.57	1.75	1.52	2.34	1.95	2.42	1.76	2.02	2.12	1.96
Jarque-Bera	5.41	7.38	6.96	3.08	4.32	13.4	3.76	10.17	11.68	6.69	6.56	3.67	5.45	1.26	3.97	1.02	5.20	4.86	4.24	3.85
Probability	0.07	0.02	0.03	0.21	0.12	0.00	0.15	0.01	0.00	0.04	0.04	0.16	0.07	0.53	0.14	0.60	0.07	0.09	0.12	0.15

Table A.3. (continued): Descriptive statistics

Private capital stock in billions of 2011 constant US Dollars (ppp)

KP	AUS	AUT	BEL	CAN	CHE	DNK	FRA	GBR	GER	GRC	IRL	ITA	JPN	NLD	NOR	NZL	PRT	SPA	SWE	USA
Mean	710	534	721	1326	709	282	3036	2597	4560	294	205	3639	5034	895	353	103	380	1781	487	12793
Median	636	533	687	1315	731	266	3227	2718	4670	293	180	3879	5163	905	395	98	364	1608	502	12126
Maximum	1575	748	992	2420	909	469	4331	3670	5742	491	433	4686	7997	1275	515	168	585	3298	669	22156
Minimum	234	243	359	434	374	115	1144	1113	2402	118	54	1839	941	383	155	48	161	649	254	4427
Std. Dev.	375	164	189	596	158	114	1027	844	1046	113	122	868	2622	276	105	34	136	849	115	5845
Skewness	0.70	-0.21	-0.19	0.16	-0.57	0.26	-0.45	-0.33	-0.52	0.12	0.69	-0.64	-0.26	-0.29	-0.46	0.36	0.04	0.47	-0.36	0.28
Kurtosis	2.51	1.74	2.01	1.88	2.17	1.73	1.91	1.63	2.02	1.81	2.27	2.11	1.50	1.94	2.00	2.14	1.71	1.94	2.18	1.77
Jarque-Bera	5.16	4.15	2.63	3.16	4.64	4.36	4.64	5.41	4.76	3.46	5.63	5.73	5.87	3.42	4.30	2.94	3.89	4.72	2.77	4.24
Probability	0.08	0.13	0.27	0.21	0.10	0.11	0.10	0.07	0.09	0.18	0.06	0.06	0.05	0.18	0.12	0.23	0.14	0.09	0.25	0.12

GDP in billions of 2011 constant US Dollars (ppp)

	AUS	AUT	BEL	CAN	CHE	DNK	FRA	GBR	GER	GRC	IRL	ITA	JPN	NLD	NOR	NZL	PRT	SPA	SWE	USA
Mean	519	233	290	850	292	174	1588	1514	2339	212	100	1495	2945	481	188	86	179	908	261	9258
Median	461	218	278	831	283	169	1565	1459	2260	207	66	1595	3186	437	179	76	174	839	252	8579
Maximum	1074	381	465	1538	454	246	2503	2538	3618	365	286	2192	4568	789	335	157	289	1590	446	16940
Minimum	165	85	111	275	139	87	540	696	973	57	23	501	575	177	60	38	49	241	112	3212
Std. Dev.	269	93	108	380	86	50	608	570	803	83	74	532	1331	195	88	34	80	412	94	4272
Skewness	0.54	0.10	0.06	0.25	0.22	-0.02	-0.07	0.30	-0.07	-0.06	0.78	-0.41	-0.36	0.18	0.19	0.59	-0.08	0.21	0.36	0.31
Kurtosis	2.08	1.77	1.83	1.86	2.11	1.65	1.79	1.75	1.72	2.29	2.20	1.83	1.67	1.73	1.67	2.11	1.60	1.81	2.02	1.72
Jarque-Bera	4.72	3.61	3.25	3.61	2.32	3.88	3.47	4.52	3.84	1.20	7.17	4.76	5.33	4.07	4.48	5.09	4.62	3.68	3.43	4.70
Probability	0.09	0.16	0.20	0.16	0.31	0.14	0.18	0.10	0.15	0.55	0.03	0.09	0.07	0.13	0.11	0.08	0.10	0.16	0.18	0.10

Working age population (15-65) in millions

	AUS	AUT	BEL	CAN	CHE	DNK	FRA	GBR	GER	GRC	IRL	ITA	JPN	NLD	NOR	NZL	PRT	SPA	SWE	USA
Mean	10.88	5.12	6.56	17.95	4.51	3.39	36.96	37.33	52.68	6.55	2.26	37.08	79.11	9.70	2.74	2.20	6.33	25.47	5.52	162.00
Median	10.91	5.17	6.67	18.25	4.50	3.44	38.24	37.28	53.51	6.61	2.12	38.35	81.06	10.15	2.71	2.15	6.51	25.45	5.43	161.00
Maximum	15.79	5.79	7.31	24.32	5.57	3.64	42.13	41.87	55.88	7.42	3.11	39.04	87.13	11.15	3.41	3.00	7.07	31.76	6.18	212.00
Minimum	6.29	4.59	5.90	10.49	3.50	2.94	29.09	34.11	48.84	5.47	1.64	32.85	59.31	7.01	2.26	1.39	5.34	19.62	4.93	109.00
Std. Dev.	2.78	0.41	0.41	4.19	0.55	0.21	3.96	2.31	2.61	0.73	0.50	2.05	7.59	1.32	0.32	0.48	0.61	3.90	0.35	31.69
Skewness	0.05	0.00	0.03	-0.21	0.15	-0.52	-0.39	0.57	-0.30	-0.17	0.46	-0.69	-0.91	-0.59	0.47	0.06	-0.35	0.13	0.44	0.00
Kurtosis	1.88	1.52	2.15	1.90	2.07	2.06	1.91	2.19	1.46	1.38	1.88	1.87	2.85	1.95	2.31	1.84	1.59	1.78	2.20	1.82
Jarque-Bera	2.95	5.14	1.69	3.25	2.23	4.56	4.21	4.52	6.35	6.42	4.95	7.38	7.74	5.85	3.14	3.18	5.82	3.63	3.34	3.23
Probability	0.23	0.08	0.43	0.20	0.33	0.10	0.12	0.10	0.04	0.04	0.08	0.02	0.02	0.05	0.21	0.20	0.05	0.16	0.19	0.20

Table A.4. Countries sample and their corresponding alpha-3 codes used in all Tables and Figures whenever it applies

Country	Country code
Australia	AUS
Austria	AUT
Belgium	BEL
Canada	CAN
Switzerland	CHE
Denmark	DNK
France	FRA
Germany	GER
Greece	GRC
Ireland	IRL
Italy	ITA
Japan	JPN
Netherlands	NLD
New Zealand	NZL
Norway	NOR
Portugal	PRT
Spain	SPA
Sweden	SWE
United Kingdom	GBR
United States	USA

Appendix B

Figure B.1 : Scatter plot elasticities of public investment fLows versus private capital stocks (*Kp, Ig*): Sample 1960-2007

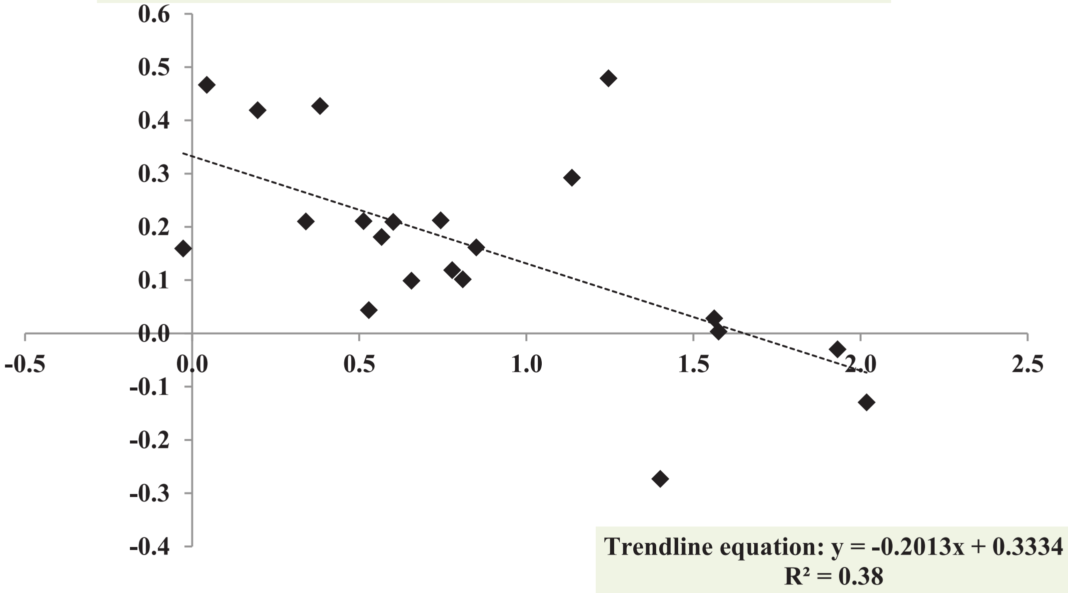


Figure B.2 : Scatter plot elasticities of public investment flows versus private capital stocks (*Kp, Ig*): Sample 1960-2015

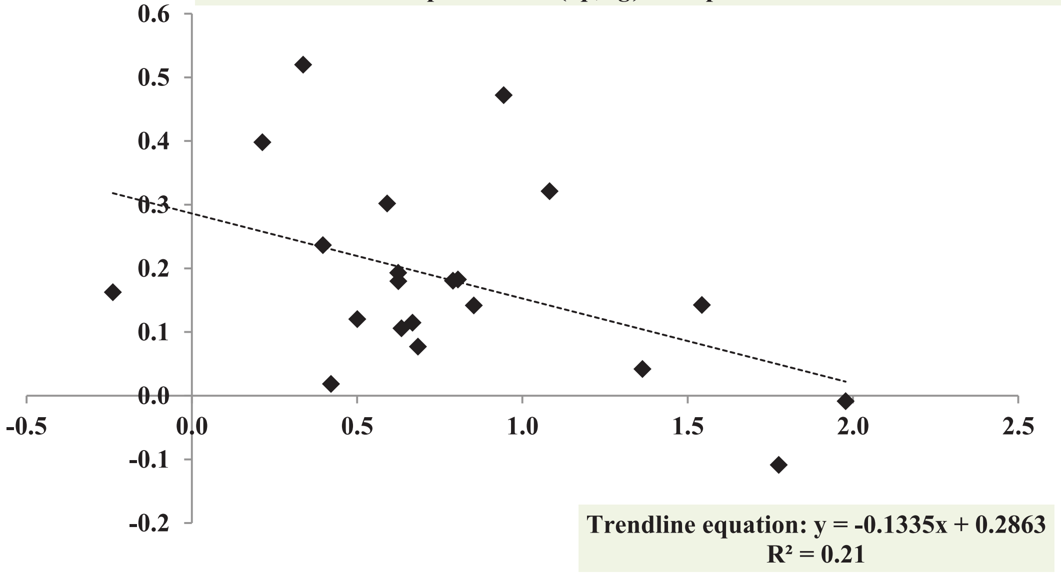


Figure B.3. Simulated (CYC) and actual (ACT) public investment flows as percent of GDP

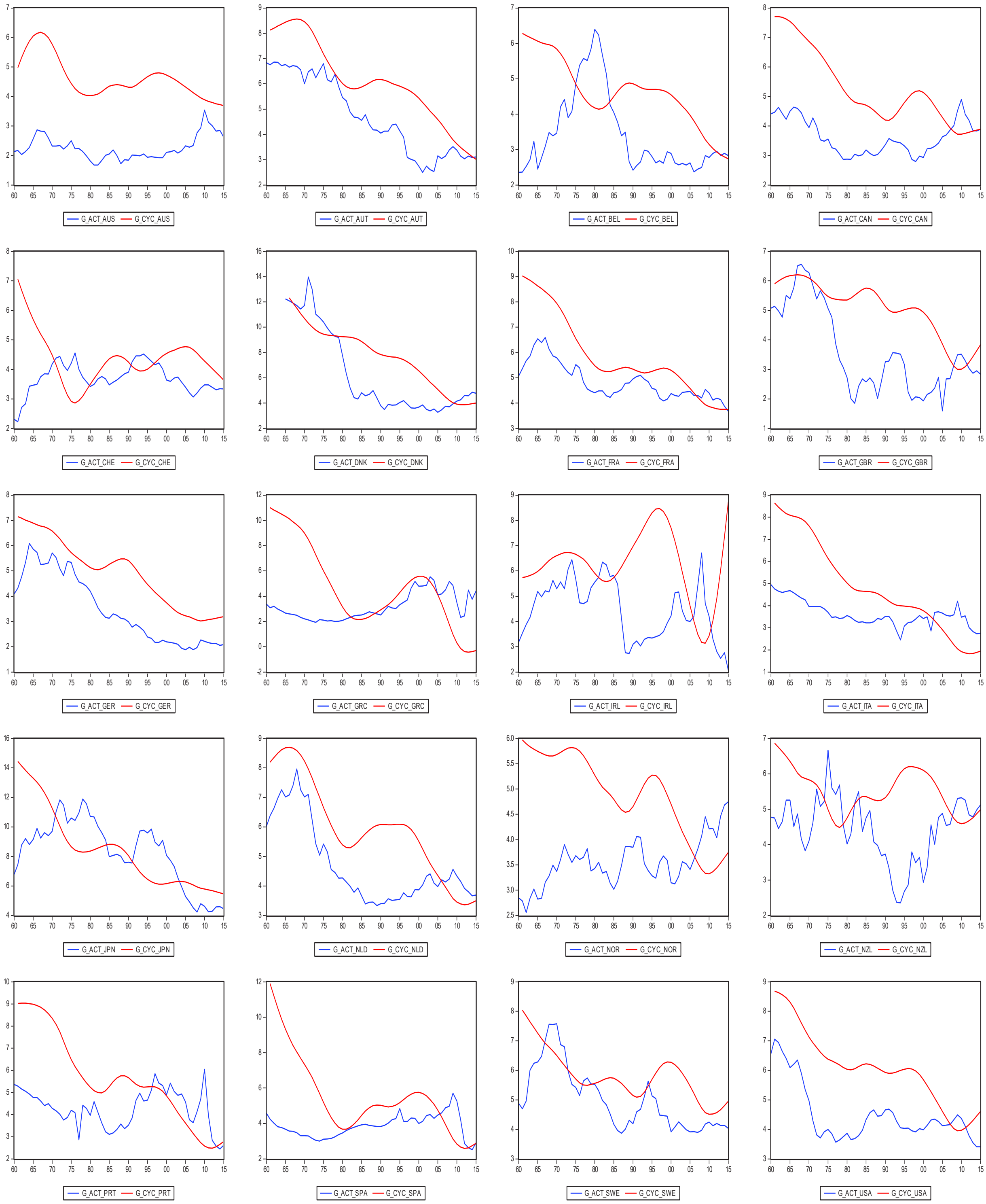


Figure B.4. simulated potential public debt calibrating differential interest rate and economic growth to the ones of the United States

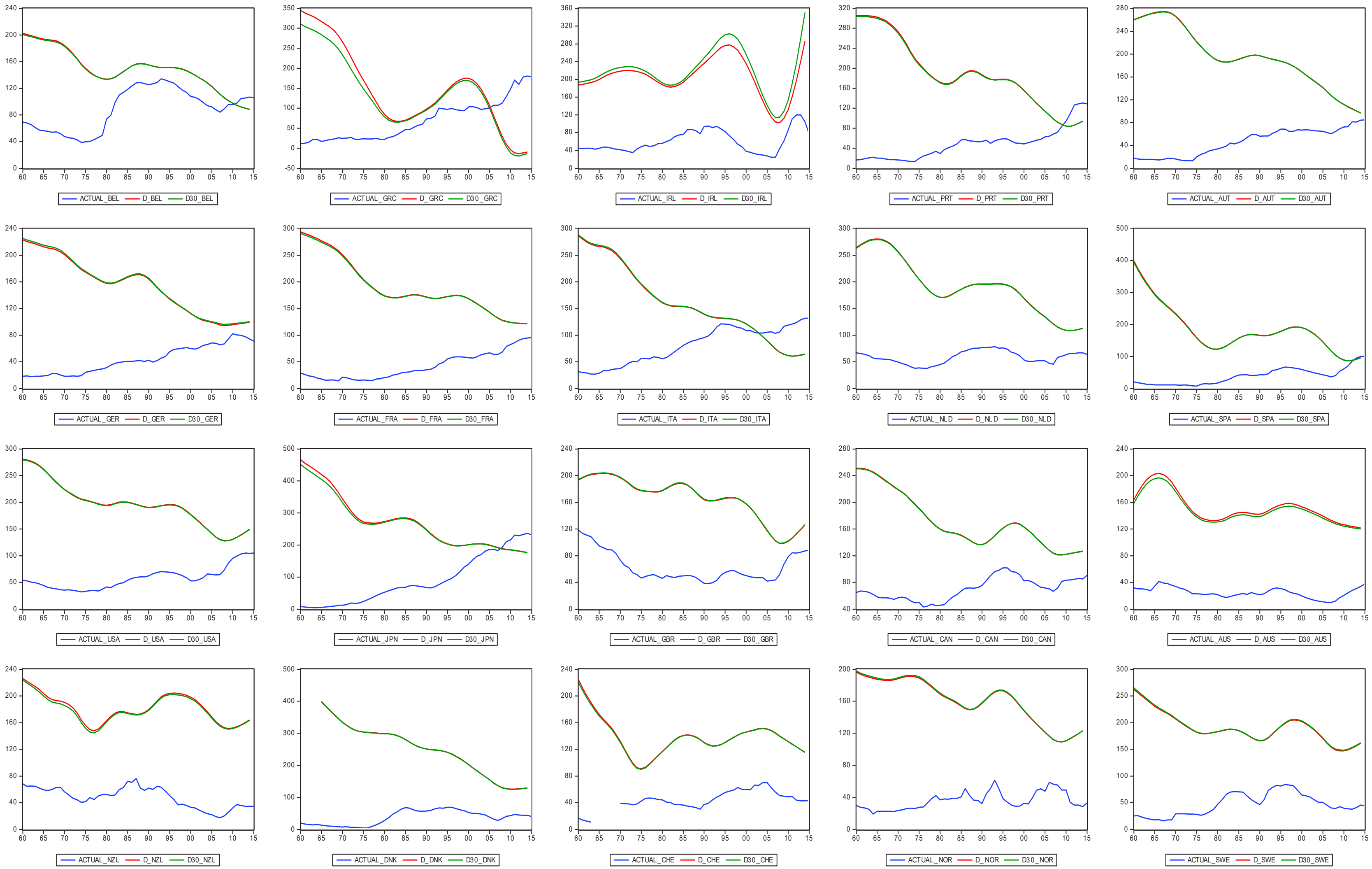


Figure B.5. Actual and simulated potential public debt corresponding to financing the potential public capital and an extra 20% of actual tax revenues