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On the public debt and growth threshold: One size does not necessarily fit all[*]

El Mostafa Bentour[†]

Abstract: In a time of high debt and sluggish economic growth, the Reinhart and Rogoff (2010) conjecture of a common 90% debt threshold for advanced economies triggered a controversial debate among economists and policymakers. We analyze the accuracy of this result for a sample of 20 advanced economies over 1880-2010. Using a regression kink model with an unknown threshold proposed by Hansen (2017), we examine the relationship between public debt and economic growth. We show that the relationship between public debt and economic growth for the whole sample changes by periods and countries samples and subject to data and country heterogeneities. The relationship is instable either by country, by group of countries or across periods of time and particularly sensitive to country size, government effectiveness and government expenditures. The kink regression method shows diverse curves for the debt-growth relationship. For a set of countries, growth slows starting from low debt levels over the postwar period. However, other countries start flourishing from low to medium levels of debt, while some countries show flat curves in the debt-growth relationship, especially over the long period 1881-2010. These findings reject the existence of any common threshold fitting all countries.

JEL Codes: C13, C15, C23, H63, O57.

Keywords: Public debt, Economic growth, Regression kink, Non-linearity, Threshold effects.

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1. Introduction

In 2010, the signals of a sovereign debt crisis in Europe and other advanced economies divided economists and policy makers towards the efficiency of the economic policies serving to get out from the recession. Some are in favor of the continuity of the stimulus packages already implemented at the beginning of the crisis, while others call for urgent fiscal consolidation and austerity policies to reduce the public deficit and debt levels. The former, believing in the role of Keynesian multipliers, make growth a priority to stabilize the deficit and debt ratios. The latter hold that high levels of debt hamper growth, which can be explained by a negative causality running from debt to economic growth. In both cases, the relationship between public debt and economic growth is more set at the forefront implying many controversies.

Thus, in such a controversial subject, Reinhart and Rogoff (2010) (RR (2010) hereafter) reported that, for a sample of 20 advanced countries, there is a 90% common threshold of the government debt-to-GDP ratio over which debt has a negative effect on economic growth. Several authors[3] have contested this result, especially after the revelation of some codification and calculation errors in the RR (2010) article.[4] This resulted in an influx of research trying different methods to study a set of econometric properties likely to alter the link between economic growth and public debt.

The trend of emerging empirical literature has been the examination of a concave non-linear relationship between debt and growth and some has focused on a variety of econometric issues, such as endogeneity, causality, and heterogeneity rather than using a proper theoretical modeling framework. Despite the use of a variety of econometric tools to overcome such issues, no consensus has been found about a robust existence of a single threshold that fit all and at which debt starts to alter growth. Moreover, despite that most researchers agree on the negative correlation between high public debt and economic growth, it is difficult to agree on the causality direction between debt and growth in the long-term as suggested by the economic theory. Many economists warn against the hasty interpretation of these researches and call for more investigation on this subject (Panizza and Presbitero, 2012; Minea and Parent, 2012).

In theory, the effects of debt on economic growth are summarized by the "conventional analysis" which reflects the dominant views among economists and policymakers (Elmendorf and

[3] See, for example, Fergusson and Johnson (2011), Herndon et al. (2013), Baglan and Yoldas (2013), Pescatori et al. (2014), Egert (2015), Eberhardt and Presbitero (2015), Chudik et al. (2017).

[4] See Herndon et al. (2013).

Mankiw, 1999). This approach adopts the deficit budget Keynesian view in the short run, assuming that a deficit financed by government bonds boosts economic activity. Government expenditures act as a stimulus for the aggregate demand in the context of Keynesian prices and wages rigidities. Nevertheless, for the short run, the debate is rather on the composition of public spending. The debt intended to finance the capital goods, would have positive short term effects, which may become negative in the long run due to the induced risk premium following high debt (Aschauer, 2000). Keynesians report that the spending effect is positive for all expenditure types, and only differs in efficiency.

In the long run, the economy adheres to the classical vision, for which government debt reduces capital stock and lower productivity, hence, reducing the output. This goes through diverse channels as Hansen (1959) reported: higher debt can trigger higher private saving, less incentives to work and invest especially for the owners of the government bonds and negative incentive effect due to additional taxes needed to pay the debt service. Likewise, public debt can crowd out private investment by reducing credit to the economy or by raising long-term interest rates (Modigliani, 1961).

However, according to Barro (1974), economic growth can be insensitive to public debt. Under the assumption of perfect information, which assumes rational expectations, economic agents expect future taxes to finance the deficit generated by the new public spending and thus reduce their expenditures. This effect is known as Ricardian equivalence[5]: any public expenditure reducing public saving is assumed offset by an increase in private saving by an equivalent amount. Consequently, the national saving is unchanged and no effect is expected on other economic variables.

If theory suggests mainly a causality running from debt to economic growth, few papers examining empirically this issue have not reached any consensus about the direction of the causality (Panizza and Presbitero, 2012; Sosvilla-Rivero, 2015 and Di Sanzo and Bella, 2015). The relationship between public debt and economic growth could also be eclipsed by the interference of other economic and institutional variables. Some authors criticized previous empirical works for the omission of such institutional variables in the debt growth analysis

[5] The name Ricardian equivalence is due to Buchanan (1976) who found a similarity between the proposal of Barro (1974) and that of David Ricardo. Moreover, O'Driscoll (1977) notes that Barro's (1974) proposition contradicts Ricardo's conclusions that there is no equivalence of choice between financing a war by taxation or a debt and decides to call it "Ricardian non-equivalence". Barro's (1974) proposal is also referred to as modern Ricardian equivalence theorem (Ahiakpor, 2013).

(Panizza, 2015; Kourtellos et al., 2013). Nevertheless, few researches tried to enrich the debt growth relationship by other macroeconomic and institutional variables (Sharpe, 2013, Pan and Wang, 2013; Greiner, 2011; Marchionne and Parekh, 2015).

Given the absence of consensus on the tolerable level of debt, questions still need to be asked: is too high public debt reduces economic growth? Does the turning point in the relationship between growth and debt exist for all countries and all times? What is its size? Does it fit all countries or is a country specific one?

The purpose of this article is to investigate the debt threshold effect existence, its size and whether it fits all or varies across countries and periods. The previous empirical studies have many shortages: those using a long period of analysis as in RR (2010) suffer from methodological issues. They generally adopt simple descriptive statistical approaches to generalize for a common threshold that fits all. While those using different econometric tools could have short time sample bias. Generally, their samples start after 1970s. The main papers set exogenous thresholds to test.

This paper adds to the existing literature by adopting a different approach. Unlike previous researches that examine the debt-growth thresholds across panels of countries, our methodology gives priority to country specific analysis. Surveying the previous empirical approaches and starts from their limitations, we use a novel econometric method proposed by Hansen (2017) that search endogenously thresholds for individual countries. We undertake estimations using long time period (1880-2010) and sub-periods depending on the World major economic and political events. We also run countries panel regressions by varying countries sample according to some sizeable countries, exchange rate regime and type of government expenditures and effectiveness.

The remainder of the paper is organized as follows. Section 2 reviews the previous literature related to the subject. Section 3 describes the methodologies used. Section 4 describes the data and descriptive statistics. Section 5 provides econometric results for country specific regressions. Section 6 presents estimation results for panel analysis. Section 7 concludes.

2. Literature review

The government debt threshold issue has been extensively studied since the 2010 debt crisis, provoking several controversies. Initiated by the early work of RR (2010), researchers examine public debt thresholds for different panel of countries correcting for econometric issues. Table 1 summarizes the main contributions.

Insert Table 1 about here

RR (2010) found that the correlation between public debt and growth is low for normal debt levels and becomes strong and negative when the debt-to-GDP ratio exceeds the 90%. They observe that median and average growth rates corresponding to debt ratios over this threshold shrink by respectively 1 and 4 points. This result, based on a simple descriptive statistical approach, has given rise to several empirical researches examining the relationship between debt and growth using increasingly econometric methods.

In a subsequent paper, Reinhart et al. (2012) emphasized their previous findings of the 90% threshold by analyzing periods of public debt overhangs for a sample of 22 advanced economies back to nineteenth century.[6] They defined a debt overhang period as a debt-to-GDP more than 90% lasting for at least five consecutive years. As a result, 26 periods were detected and 23 of these are associated with lower growth. On average, an annual growth lower by 1.2 percentage point than in periods of debt ratios less than 90%. However, from the 22 advanced countries, only 13 have episodes of debt overhang from which two countries (Italy and Greece) have both 8 periods of debt overhang (4 each). Therefore, the sample of countries, with different economic policy experiences, used to emphasize the 90% common rule is reduced. Furthermore, almost all episodes of high debt resulted from costly wars and the Great Depression and, only six countries have debt overhang in peacetime: Belgium, Canada, Greece, Ireland, Italy and Japan. So, should this small sample of heterogeneous countries in size, time sample and monetary sovereignty be sufficient for concluding about the common 90% threshold?

In fact, the conjecture of a common threshold does not seem to be accepted by many authors. For example, Ferguson and Johnson (2011) stated that RR (2009, 2010) "*jumble big and small countries together from different areas and different political choices. This makes induced policy lessons from such samples a likely misleading exercise*". They argue that *"political choices for smaller countries are frequently influenced by external factors, while big countries like United States and Japan are the principal players in the international system of which policies affect the rest of the world"*. Consequently, the authors opposed the idea of a common threshold debt

[6] The time sample differs between countries depending on data availability. US have the largest dataset (1791-2011) while Ireland has the shortest sample (1924-2011).

arguing by historical counter-examples. Especially, the United Kingdom constitutes an interesting fact against Reinhart and Rogoff claim showing that the British industrial revolution flourished while debt-to-GDP ratio exceeds 200% for decades.

Empirically, important controversy came from Herndon et al. (2013). By replicating the exercise of RR (2010), these authors correct some data processing errors. Consequently, the average growth rate of countries with government debt-to-GDP ratios more than 90% is not dramatically different from that calculated for countries with moderate to high debt-to-GDP ratios. This paper triggered an influx of researches examining debt thresholds using different econometric tools (Table 1).

An important number of these researches replicate the same sample of the 20 advanced countries of RR (2010) and find different results that generally pointed out to low levels of debt thresholds. Baglan and Yoldas (2013) used Bayesian inference and found a debt threshold of 20%. Egert (2015) used the Hansen (1999) method and found a debt threshold of 20 to 30%. Lee et al. (2017) examining the relationship between public debt and median GDP growth suggest that the debt threshold may exist around 30%. Surprisingly, Minea and Parent (2012), using Panel Smooth Threshold Regression (PSTR) models found a convex relationship between debt and growth where the effect is negative below a high debt threshold of 115% and positive above this threshold. Even though, these results are surrounded by more uncertainties and may result in dangerous consequences when translated into policy decisions.

Other researchers used different samples and periods to study the long run effects of debt on economic growth. For example, Pescatori et al. (2014) analyzed debt and growth data over long history considering lead economic growth by 1, 5, 10 and 15 years to be affected by the current debt. Their analysis rejects any threshold from which economic growth is undermined. However, they find that high debt increases output volatility. They also find that countries with high but declining debt grow as fast as countries with lower growth. Eberhardt and Presbitero (2015) study nonlinearity by correcting the heterogeneity of the debt-growth relationship across countries. Their results highlight negative non-robust relationship between public debt and long-term economic growth, but failed to sort out a common debt threshold for all countries. Chudik et al. (2017) developed tests for threshold effects in the context of dynamic heterogeneous panel data models and find no evidence for a universally applicable threshold effect. Regardless of the threshold, they find significant negative long-run effects of public debt build-up on output growth. Furthermore, Syssoyeva-Masson and De Sousa Andrade (2017) highlighted the long memory of public debt series and recommend studying the debt-growth relationship in a long run framework.

Nevertheless, other researchers seem to support RR (2010) findings, although not necessarily the 90% debt threshold. Caner et al. (2010) were the first to review the results of RR (2010) and confirm the negative link between public debt and economic growth above the 90% threshold. Kumar and Woo (2010) also highlighted a negative non-linear relationship for higher debt levels for a sample of advanced and emerging countries over a period of 1970-2007. Particularly, the per capita growth is 0.2% lower following a 10-percentage point increase in the debt ratio. Lin (2014) applied a threshold quantile Lasso regression to a sample of 62 cross-sections combining developing and developed countries over the period 1991-2005, and to individual countries for a subsample of data of 22 countries over the period 1961-2010. For cross-country analysis, he confirms the 90% threshold for the median quintile as defended by RR (2010). However, for country-specific analysis, he shows that tipping points widely range between 10% and 100% across countries. The widespread values of threshold were emphasized especially when controlling for macroeconomic and demographic changes. The existence and value of these thresholds increase by quintile reflecting asymmetric effects of debt on growth, and are more common in developing than in developed countries. Checherita and Rother (2010) found a significant polynomial term between public debt and per capita economic growth considering a sample of 12 Eurozone countries over the period 1970-2009. Unlike researches that pointed to a defined threshold, their paper indicates an interval (90-100%) from which public debt starts to hamper economic growth.

3. Econometric Methodology

Unlike the previous researches that examined essentially panel groups, and a few of them weakly investigated country specific regressions[7], we adopt a different approach in which we give more importance to country specific regressions. For this purpose, we first run an innovative methodology namely a regression kink recently developed by Hansen (2017). This method is more appropriate as it permit searching endogenously for unknown thresholds.

[7] Gomez-Puig and Sosvilla-Rivero (2017) use time series regressions for 11 Eurozone countries between 1961 and 2015 and conclude for a nearly threshold that vary from a minimum of 21% in France to a maximum of 61% in Belgium.

To support our results, we also run panel regressions in which we assume that economic growth is a non-linear[8] (quadratic) form of debt. Our aim, by this second approach, is to show that we could find thresholds in the relationship between debt and growth in heterogeneous panel groups as shown by partisans of RR (2010) results, but these are instable by time and countries sample as well as other institutional characteristics.

Our approach for panel regressions is different from the previous researches as it considers long period of analysis split according to the major events in the international economic and political order, as well as, varying sample by country size and level of public expenditures and government effectiveness. This approach complements the first one in results: while the individual regressions show different relationship curves by countries that are also instable over time, the panel regressions show that threshold is highly affected by country sample and time period. Both approaches argue against a unique threshold that fit all countries.

3.1. Country specific methodology

For the individual regressions, we use the kink regression method developed by Hansen (2017) that searches for endogenous thresholds.[9] The regression function is everywhere continuous except on this threshold where the slope has a discontinuity. Instead of assuming exogenous known thresholds as in many previous empirical researches and by the traditional regression discontinuity models, this method considers that the threshold is unknown and should be estimated.

The recent regression kink with an unknown threshold constitutes an important advancement of the threshold regression models. The first class of such models were regression discontinuity design (RDD) early introduced by Thistlethwaite and Campbell (1960) and recently enhanced by regression kink design (RKD) (Nielsen et al., 2010) and emphasized theoretically by Card et al. (2012). Both RDD and RKD are especially involved when a policy variable of interest (the outcome) is totally or partially determined by a known assignment rule of an observed treatment variable (covariate). Both methods become important for identifying causal effects in

[8] The example of debt-growth non-linear effect is becoming more used to test new econometric methods. See, for example, Egert (2015), Henderson et al. (2015) or Hansen (2017).

[9] The difference in countries in terms of institutions, governance, economic policies, etc., contributes mainly to such endogeneity bias. In general, these variables are difficult to measure, and their effects could be better assessed in a theoretical model (such as endogenous growth model) rather than in a simple non-linear relationship for which the main goal here is to detect a turning point in the link between debt and growth simply from the data generating process (DGP).

observations settings in many areas like educational outcomes, election outcomes, unemployment, etc., (Card et al., 2017).[10] The only difference is that the RDD uses a "discontinuity" or a "jump" in level of a treatment status at a threshold of an assignment variable, while RKD examines discontinuities in derivatives (slope discontinuities) rather than the level (Athey and Imbens, 2017). Despite their important use, some authors warn that their results could be biased especially for smaller population size particularly in the presence of confounding nonlinearities between an assignment variable and an outcome variable (Ando, 2017).

As for the Hansen (2017)'s regression kink with an unknown threshold, it is the latest method to determine endogenously thresholds without need for a treatment or an assignment variable as in the previous methods. The conventional regression kink design assumes that the threshold is known. This is suitable in many policy-oriented applications where the threshold is determined by policy (Hansen, 2017). Instead, we treat the threshold as an unknown to be estimated. This method is particularly appropriate when the threshold is either not set by the policy, or when wishing to investigate the robustness of this assumption. The features of such regression correspond highly with our aim of examining endogenous thresholds from a direct relationship between economic growth and public debt. We aim rather to confirm or deny the existence of debt thresholds than measuring any policy effects as the previous methods do. Hansen (2017)[11] developed an inference and estimation toolkit that tests for the presence of the threshold, estimation and inference on the regression function and parameters.

Using $(a)_- = min[a, 0]$ and $(a)_+ = max[a, 0]$ to denote the "negative part" and "positive part" of a real number a, the Hansen's regression kink model takes the following form:

$$g_t = \beta_1(d_t - \gamma)_- + \beta_2(d_t - \gamma)_+ + \beta_3' h_t + \varepsilon_t \tag{1}$$

Where (g_t, d_t, h_t) are, respectively, variables describing economic growth, public debt-to-GDP ratio, and a k-vector of other explanatory variables which includes an intercept. ε_t is the error term independent and identically distributed with zero mean and constant variance $\left(\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)\right)$. The variables (g_t, d_t, h_t) are observed for $t = 1, \dots, n$. The parameters to be estimated are the regression slopes β_i, with $i = 1,2,3$, and the parameter γ called the threshold or "kink point". In equation (1) the slope with respect to the variable d equals β_1 for values of d_t

[10] Other threshold models include a variety of autoregressive time series models with threshold used particularly in financial applications (Chen et al., 2011).
[11] A theoretical generalization of Hansen (2017) method to panel data recently appeared in Zhang (2017).

less than γ, and equals β_2 for values of d_t greater than γ, yet the regression function is continuous in variables d and h, except the slope with respect to d which is discontinuous at $d = \gamma$ (kink point). Equation (1) has $p = 3 + k$ parameters. $\beta = (\beta_1, \beta_2, \beta_3)$ are the regression slopes and are generally unconstrained so that $\beta \in R^{p-1}$. However, for the parameter γ, the model only makes sense if the threshold is in the interior of the support of the threshold variable d. We thus assume that $\gamma \in \Gamma$ where Γ is compact and strictly in the interior of the support of d.

To be applied to the debt growth relationship, we rewrite equation (1) with lagged independent variable d_{t-1} (so that this is plausibly pre-determined) and set $h_t = (g_{t-1}, 1)$ and then $\beta_3 = (\delta, c)$ so that the regression contains a lagged dependent variable to account for dynamic effects and minimize autocorrelations. Equation (1) becomes:

$$g_t = \beta_1(d_{t-1} - \gamma)_- + \beta_2(d_{t-1} - \gamma)_+ + \delta g_{t-1} + c + \varepsilon_t \quad (2)$$

Equation (2) can be written as $g_t = \beta' x_t(\gamma) + \varepsilon_t$, where $x_t = ((d_t - \gamma)_-, (d_t - \gamma)_+, g_{t-1})'$ and the least squares criterion for estimation is:

$$S_n(\beta, \gamma) = \frac{1}{n}\sum_{t=1}^{n}\left(g_t - \beta' x_t(\gamma)\right)^2 \quad (3)$$

Minimizing (3) yields the least squares estimator:

$$\left(\hat{\beta}, \hat{\gamma}\right) = arg \min_{\beta \in R^{k-1}, \gamma \in \Gamma} \{S_n(\beta, \gamma)\} \quad (4)$$

The criterion $S_n(\beta, \gamma)$ is quadratic in β but non-convex in γ. Hansen (2017) uses a combination of concentration and grid search. Particularly, by concentration we write:

$$\hat{\gamma} = \arg \min_{\gamma \in \Gamma} \min_{\beta \in R^{k-1}} \{S_n(\beta, \gamma)\} = arg \min_{\gamma \in \Gamma} \{S_n(\hat{\beta}(\gamma), \gamma)\} = \frac{1}{n}\sum_{t=1}^{n}\left(g_t - \beta' x_t(\gamma)\right)^2 \quad (5)$$

Where, for a given γ, the parameters $\hat{\beta}(\gamma)$ are the least squares coefficients from regressing g_t on $x_t(\gamma)$. The kink point $\hat{\gamma}$ is determined by a grid search over $\gamma \in \Gamma$, and once found, the parameters $\hat{\beta}$ are determined by standard least squares of g_t on $x_t(\hat{\gamma})$. The deduced regression function is then:

$$g_t = \hat{\beta}' x_t(\hat{\gamma}) + \hat{e}_t \quad (6)$$

Where $\hat{e}_t$ are residuals with an estimated error variance:

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{t=1}^{n} \hat{e}_t^2 = S_n(\hat{\beta}(\gamma), \gamma) \quad (7)$$

In order to test for the unknown threshold, Hansen (2017) conducted an algorithm[12] to test the nested regression model (2) against the following linear model assuming $\beta_1 = \beta_2 = \beta_l$:

[12] The algorithm is presented in the appendix C (algorithm 1).

$$g_t = \beta_l d_{t-1} + \delta g_{t-1} + c + \varepsilon_t \quad (8)$$

3.2. Panel methodology

To investigate the existence of a threshold at which debt reduces growth, we proceed for the panel approach by the same way as in Checherita and Rother (2010).[13] We estimate the following equation:

$$g_{i,t+5} = \alpha d_{i,t} + \beta d_{i,t}^2 + \gamma_i + \varepsilon_{i,t} \quad (9)$$

Where $g_{i,t+5}$ is the 5-years lead economic growth for country i; $d_{i,t}$ is the debt to GDP ratio for country i at time t, α and β are parameters associated with the debt and its square, γ_i is the constant term associated with each country i called fixed effect, and $\varepsilon_{i,t}$ is the error term with zero mean and constant variance ($\varepsilon_{i,t} \sim iid(0, \sigma_\varepsilon^2)$). The time lag of five years is assumed: the current debt will affect growth after 5 years. This is the case where debt is more negatively correlated with growth (Table 1.A).

This equation is analogous to many modelling curves in the economic literature: Mincer (1974)'s earning equation, Laffer curve (tax rate, Government revenue), Kuznets curve (income, inequality), where the dependent variable is a quadratic polynomial form of the explanatory variable and thus expected to have a turning point (negative slope of the quadratic term). Similarly, the non-linear term in (9) assumes that the rhythm by which debt affects growth changes from a specific turning point. For example, low public debt could have a positive effect on growth and starting from a certain threshold (high debt), the effect becomes negative.

Equation (9) changes its trend if it has a derivative null at a certain level of debt ratio. The debt threshold $\hat{d}$ is then deduced by deriving (9) according to the debt ratio:

$$\hat{d} = -\frac{\alpha}{2\beta} \quad (10)$$

Equation (9) is to be estimated using Generalized Least Squares (GLS) method. However, to remedy to the major problems raised by the literature in terms of endogeneity, which may be caused especially from the omission of other explanatory variables or from reverse causality, the generalized method of moments (GMM) involving instrumental variables is invoked.

[13] Our exercise distinct from Checherita and Rother (2010) by the long span time considered the sample of countries and different instrumental variables. For example, these authors use Gross Capital Formation as instrument while it is rejected in our choice. The Pearson's correlations (Table A.3) reveal that this variable is weakly correlated with the explanatory variable (public debt).

Thus, the equation to be estimated by GMM is:

$$g_{i,t+5} = \alpha d_{i,t} + \beta d_{i,t}^2 + \varphi Inst + \gamma_i + \varepsilon_{i,t} \quad (11)$$

Where $Inst$ are a set of instrumental variables and φ the set of their associated parameters. The threshold is deduced as in (10).

Another interesting method used previously by some authors is the PSTR models standing for Panel Smooth Transition Regression. Gonzalez et al. (2005) developed these models as an enhancement for the Panel Threshold Regression (PTR) models of Hansen (1999). These models explain the dependent variable on a linear term of the independent variable augmented with non-linear terms as a multiplication of the same independent variable with an indicator function. This latter is modelled in the form of a logistic function which depends on a threshold variable. In case the threshold variable is the same as the dependent variable, this yields a polynomial form as in (9). Despite their popularity and advantages of accounting for fixed effects in the panel data, the PSTR do not allow lagged explained variables to be in the right-hand side of the specification (Colletaz and Hurlin, 2012). This means they do not allow for dynamic effects.

4. Data and preliminary analysis

In this section, we present the data sources and a preliminary analysis (as the Pearson's correlations and statistical heterogeneity tests) for the data generating process.

4.1. Data description

Regarding this work, we consider a sample of 20 developed countries over the period 1880-2010 from the IMF database. This is the same sample of countries used by RR (2010) in addition to many other subsequent researches. The methodology and description of the data are published in Abbas et al. (2010).[14] The database reports the ratio of public debt-to-GDP. The latter is from the Maddison[15] data according to the Geary-Khamiss method, in international dollar. For consistency, the Maddison source is also used for the GDP growth.

Our sample counts for twenty advanced countries, namely: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.

[14] Database and the paper are drawn from www.imf.org/external/pubs/cat/longres.aspx?sk=24332.
[15] http://www.ggdc.net/maddison/maddison-project/data.htm.

4.2. Correlations and data heterogeneity tests

Table A.1 (appendix A) shows that Pearson's correlations tend to be negative, especially between current debt and 5-years lead growth. 13 out of 20 correlations are significantly negative over 1950-2008. But, the weak correlations may suggest that the form of the relationship is not necessarily linear, since the Pearson correlations assume linearity. This seems to be revealed by scatter plots for individual countries.

An issue related to data and country samples is heterogeneity. The source of the heterogeneity could be unobserved characteristics due to other variables involving economic policies and institutions. The heterogeneity in country behaviors is known as fixed effect in econometrics. Some of the previous works have warned against the high heterogeneity of the countries due to differences in fiscal and monetary policies, country size, and quality of institutions.[16]

However, no paper has invoked the issue of data heterogeneity. We take the opportunity to study such heterogeneity. RR (2010), and other researchers, used descriptive statistics based on conditional means and medians to conclude, for all countries, the existence of a common debt threshold beyond which GDP growth slows. This assumes that the data generating process is homogenous for all countries. However, running appropriate statistical tests for the equality of means, equality of medians and equality of variances across countries, for the economic growth categorized by public debt, highly reject the null hypothesis of equality of such statistics. This means that the data generating process is highly heterogeneous across countries. Table A.2 presents the results of the equality tests for different periods. The source of variation is many times higher in between than within countries. The heterogeneity seems to decrease as the period shortens. Specifically, over 1991-2008, the null hypothesis of equality of means and medians is accepted at 5% level and the test indicates variances homogeneity.

5. Country specific analysis

The aim of this section is to analyze the relationship between public debt and economic growth and investigate whether a unique debt turning point exists for all countries. Surveyed empirical

[16] Despite its importance, we prefer not enriching equation (9) for panel data by a set of variables representing economic and monetary policies for two reasons. First, these variables are not observed over the same period length of debt and growth. Second, this could deviate the model from its assumed non-linear form leading to many ad-hoc relationships between variables. What we have done next is splitting the sample of the 20 countries and run our regressions on subsamples according to their levels in the main types of government expenditures. The level of expenditures should reflect to some extend the economic governance of the countries.

studies have not reached any consensus about a clear relationship between debt and growth. Despite the diversity of methods, they show sensitivity to different econometric problems and data samples shortness. Those using a long period of analysis as in Reinhart and Rogoff (2010, 2012) could have methodological issues, by adopting simple descriptive statistical approach to generalize for a common rule that fits all. However, those using somehow advanced econometric analysis could have short time sample bias. Generally, their samples start after 1970s. Our aim is to reconsider an investigation of the debt-growth relationship, starting from a data descriptive approach and ending to econometric estimations over a long period. We try to remedy to limitations of samples and econometric methods used in the previous literature by using both advanced recent econometric tools and rich statistical data analysis over long period for panel and country specific analyses.

5.1. Scatter Plots analysis

Despite the diversity of the econometric methods used to study the growth-debt relationship, these are generally applied to samples starting from the seventies. This coincides with the end of the Bretton Woods system and the beginning of the market liberalization in developed countries. To remedy to this shortcoming, we extend the analysis to the long period 1880-2008, split to five sub-periods corresponding to the main changes in the international economic and political order (Rodrik, 2011; Obstfeld and Taylor, 2002).[17] Theses world events may affect the stability of the macroeconomic aggregates and their interdependencies, hence the debt and growth linkages. We distinguish the following sub-periods:

- 1880-1913. This period fits with the end of the first globalization (mercantilism era);
- 1914-1945. A period with two devastating wars and the 1929 great depression. The international economic order was marked by the gold standard regime;
- 1946-1970. The world experienced strong growth and development during this period, notably with the Bretton Woods agreements and fixed exchange rate regimes;
- 1971-1990. A period of turbulent economic and political events with the end of the convertibility of the dollar in 1971 and the trend to floating exchange rates, the oil shocks

[17] Breakpoint tests based on Dickey-Fuller unit root test emphasized this partition around these dates. For example, break dates in debt series are 1918, 1941, 1964 and 2007 for USA; 1915, 1950, 1969 and 1990 for UK; 1906, 1944, 1975 and 1996 for Japan; 1896, 1960 and 1991 for France; 1902, 1937, 1974 and 1992 for Germany. Detailed results for all countries are available upon request from the author.

of the 1970s with the coexistence of unemployment and inflation and the sovereign debt crises of the 1980s;

- 1991-2008. Countries underwent extensive financial liberalization under the Washington Consensus recommendations, resulting in financial instability for many emerging markets. The World Trade Organization (WTO) was created, bilateral and multilateral free trade agreements proliferated, and the Euro currency was launched.

We first conduct a scatter plots analysis for the relationship between public debt and growth for individual countries (Figures B.1 to B.9). The relationship is plotted over eight periods; the whole period 1880-2008, the five previous sub-periods, and two other periods (1946-2008 and 1971-2008). Scatter plots are organized for each period for individual countries. We draw scatter plots for 5-years[18] lead economic growth explained by the current public debt. The chart analysis is supported by the Pearson's correlations in Table A.1 for the debt and growth over different eras.

We tend to assume lag effect between debt and growth. We believe that a deficit financed by a government debt will act with a certain delay on economic growth either on the short run or in the long run. For example, capital expenditures, which are believed to affect economic growth more than other expenditures are likely to impact growth with a delay. For example, a port or a road financed by bond issuance will make time to be constructed and begins to benefit to the community. Furthermore, other macroeconomic channels by which debt affects economic growth, such as interest rates, openness, population and others, are likely to react with a delay rather than immediate effect. Our statistical data endorsed such delay in the effect of debt on growth where current debt-to-GDP ratio is more correlated with lead growth than current growth (Table A.1).

The individual scatter plot analysis show that the relationship form changes by both countries and periods. We notably distinguish:

- Flat curves for the case of Austria and Denmark (1880-2008), Portugal (1946-1970) and Austria (1991-2008);
- Negative linear as in Germany (1880-2008 and 1880-1913), Netherlands (1914-1945), Canada (1971-1990) and Austria and Italy (1946-1990);

[18] The 5 years' lag is justified by the fact that the debt is supposed to affect growth over medium to long-run.

- Positive linear for Belgium (1880-2008), Australia and Portugal (1880-1913), Austria and Germany (1914-1945), France and Switzerland (1946-1970), Ireland and Portugal (1991-2008) and France (1946-1990);
- Convex relationship in Denmark and Norway (1880-1913), Germany and Greece (1946-1970), Italy and USA (1991-2008), Italy, Portugal, Greece, Sweden, United Kingdom and Austria (1946-1970) and Japan, Germany, USA and Greece (1946-1990);
- Concave relationship as in France, Italy, Switzerland and New Zealand (1880-2013), Australia, Canada, Italy, Spain and United Kingdom (1914-1945), Netherlands, New Zealand and United Kingdom (1946-1970), Belgium, Denmark, France, Spain, Sweden and Switzerland (1971-1990), France, Norway and Sweden (1991-2008), Belgium, France, Spain and Denmark (1946-2008) and Australia and Sweden (1946-1990).

Scatter plot analysis highlights that the form of the debt-growth relationship is rather country specific and might change by time. The economic explanation is straightforward. We note especially differences across countries in the economic and political governance, the structure of debt (external or domestic, currency of denomination, maturity), the aging population (Germany and Japan versus Portugal and Spain), the degree of openness, the size of the economy (Greece and Ireland versus Japan and United States), the structure of public expenditures, changeable economic policies and political and external environment as well as the level of the development in a country over time. Therefore, the fact that there is neither a unique economic policy, nor a comparable level of institutional and demographic variables across countries, makes less defendable "the one size debt threshold that fits all" conjecture.

5.2 Regression Kink Results

We use the kink regression method of Hansen (2017), allowing detecting unknown thresholds. Country specific regressions emphasized what is reported on country specific scatter plots. The regression kink with an unknown threshold shows a variety of forms for growth-debt relationship. Tables 2 and 3 present the regression kink results for respectively the whole and the postwar period.

Furthermore, for a better visualization of the results, we present in Figures 1 to 5, for each country and each period, a graph of three components: the first part (left) draws simply the debt ratio variable over time. The second (middle) shows regression scatter plots where the red point corresponds to the turning point (kink point or threshold) along with the regression line corresponding to equation (2). The third one (right) presents the threshold parameter in axis with the F-statistic showing the minimum of the Fisher test that indicates the threshold. Asymptotic values (confidence intervals) are displayed in dashed blue lines. All the graphs are spread over 5 landscape pages where each page contains 4 countries results for two periods: 1881-2010 and 1950-2010.

The threshold F-statistic is calculated according to the test of the hypothesis $H_0: \gamma = \gamma_0$ against $H_0: \gamma \neq \gamma_0$. The criterion test is to reject for high values of the F-type statistic $F_n(\gamma_0)$, where $F_n(\gamma) = \frac{n(\hat{\sigma}^2(\gamma) - \hat{\sigma}^2)}{\hat{\sigma}^2}$, and $\hat{\sigma}^2 = \frac{1}{n}\sum_{t=1}^{n} \hat{e}_t^2$ is from equation (7). The threshold $\hat{\gamma}$ corresponds to the minimum of the threshold F statistics which is also the minimum of the least squares criterion. Hansen (2017) presents an algorithm for the bootstrap confidence intervals for parameters and the F statistic.[19]

Insert Tables 2 and 3 about here

Insert Figures 1 to 5 about here

According to these results, we can distinguish many cases depending on the sign and magnitude of the parameters β_1 and β_2:

- ✓ Case 1: $\beta_2 > \beta_1 > 0$

  The growth rate is always positively affected by debt and the effect becomes higher after the debt threshold. This case is only supported by Australia over 1956-2010.

- ✓ Case 2: $\beta_1 > \beta_2 > 0$

  The effect of debt over growth is always positive but is reduced for debt values above threshold compared to the effect of debt values below threshold. This case is reported in some countries as Ireland (1929-2010) and Norway (1881-2010).

- ✓ Case 3: $\beta_2 > 0 > \beta_1$

  The effect is negative for debt ratios below the kink point then becomes positive after that point. This case is presented by Austria (1956-2010), Denmark, France, Italy, Spain, Sweden and Switzerland over long period 1881-2010.

- ✓ Case 4: $\beta_1 > 0 > \beta_2$

[19] The algorithm is presented in the appendix C (algorithm 2).

The effect is positive then becomes negative after the turning point. This case is showed by UK (1956-2010), US (1791-2010), Austria (1956-2010), Germany (1881-2010) and Netherlands (1956-2010).

- ✓ Case 5: $\beta_1 < \beta_2 < 0$

The effect is always negative but less emphasized after the turning point. This case is observed in Japan over both long and short periods and Italy over the recent period.

- ✓ Case 6: $\beta_1 \cong 0$ (respectively $\beta_2 \cong 0$) and $\beta_2 \neq 0$ (respectively $\beta_1 \neq 0$)

Growth is insensitive to debt ratio before the threshold (respectively after the threshold) and the effect has the sign of β_2 (respectively β_1). These special cases are showed by Ireland (1956-2010), Portugal (1881-2010 and 1956-2010) and Norway 1881-2010).

- ✓ Case 7: Flat curves

Debt neutrality is shown for the case of US and UK (1881-2010), Australia (1910-2010), Canada (1881-2010), Denmark (1956-2010), Sweden (1881-2010) and Greece (1884-2010).

The majority of the results are inconclusive when considering the confidence intervals. The threshold confidence intervals (dashed blue lines) are too large for many countries, which makes the results not strongly conclusive about precise turning points. However, besides the accuracy and precision of the results, the method emphasizes the differences in the data generating process of the debt-growth relationship across countries. It reveals that the thesis of a debt threshold is case-specific and is changeable over time. It suggests also that there is no simple formula determining a specified debt threshold or even a range of it, considered a dangerous zone not to reach, just by considering the direct relationship between the economic growth and the public debt to GDP ratio.

6. Panel specification analysis

We present in this section, scatter plots analysis and estimation results for the panel groups.

6.1 Scatter plots analysis

Figures 6 and 7 present scatter plots for the panel analysis between 5-year lead economic growth and public debt ratios. Figures are presented in panel graphs by period for the whole and the euro zone sample. These scatter plots show that economic growth is weakly correlated with the public debt. The trend-line of the relationship is flat in almost all periods for both the whole sample and the eurozone. The relationship is only apparently negative for the whole sample over 1950-2008 and the postwar period (1946-1970). For the euro zone sample, the analysis is nearly the same except a weak positive correlation over the period 1991-2008.

Insert Figures 6 and 7 about here

6.2 Panel regressions

We first estimate equation (9) using GLS method. However, for the GMM method, a set of convenient instrumental variables is needed. The choice of instrumental variables for the GMM method can be challenging. In practice, these are chosen to be correlated with the explanatory variables and orthogonal to the error term before introducing instruments (error in equation (9)), which means weakly correlated to the dependent variable of the initial regression before considering such instruments. We choose a set of variables that can act as instruments based on the Pearson's correlations with the explained and explanatory variables (Table A.3). The following variables and their first and second lags are considered to be candidates: old people dependency ratio (ODR), shares to GDP of, respectively, exports (EX), imports (IM), government consumption (GC) and gross capital formation (GCF). The source of the dependency ratio is the World Development Indicators of the World Bank, while all other instruments are from the Penn World Tables (version 8.1), which provides data back to 1950 for all the countries of the sample adjusted for purchasing power parity (PPP).

Table A.3 shows that, first, gross capital formation is weakly correlated with 5-years lead growth but also weakly correlated with debt. As a result, it can be moved away. Second, by period, exports, imports, dependency ratio and government consumption are correlated with the debt variable and weakly correlated with growth. We also used the Sargan test which excluded the GCF at the estimation stage.[20]

Equation (9) is estimated over the periods: 1950-1970, 1971-1990, 1991-2008, 1971-2008 and 1950-2008. The periods 1880-1913 and 1914-1945 were excluded as the debt series experienced breaks for the majority of countries during these periods due to the great depression and the World wars. Similarly, estimations start from 1950 instead of 1946 as the debt of many advanced countries stands highly abnormal following World War II. Reinhart and Rogoff (2009) reported

[20] This statistic follows a χ^2_{r-k} distribution where r is the number of instruments and k is the number of estimated parameters (including the constant term).

that defaults and restructuring debt in these times are the highest in history.[21] For the similar reason, the recent financial crisis is not considered in estimation. The main argument is that these points at the end of the period could statistically distort the results in addition to the ambiguous character of the crisis on debt and growth.[22]

Equation (11) is estimated with fixed effect[23] relative to each country γ_i. Some authors remove the fixed effect by differentiating the model. However, the fixed effect is important to keep in our case as it considers the heterogeneity of the panel. If the heterogeneity is rejected, then we can remove the fixed effects. Nevertheless, differencing the equation will modify the assumed non-linear quadratic form and not allow to easily deduce the concavity of the relation and thus the value of the threshold according to the formula (10). We also run a variety of tests for no cross-section dependence[24] for the estimated panel model over all the periods. Table A.6 summarizes results for the 20 OECD sample countries and the 10 Euro sample countries. These tests highly reject the null hypothesis of no cross-section dependence. Therefore, heterogeneity and cross-section dependence among other problems facing panel methodology support our approach considering studying growth-debt relationship for countries individually.[25]

Table 4 presents estimation results by GLS and GMM methods for five periods; 1950-1970, 1971-1990, 1991-2008, 1971-2008 and 1950-2008. The results are not significant for the GLS over all periods except the whole period 1950-2008 over which, the form of the equation is convex ($\beta > 0$) and the threshold provided means that debt start to enhance growth when debt ratio crosses the threshold.

However, the GMM method leads to significant results. For the fixed effects model, two thresholds result from the concavity of the relation ($\beta < 0$) over the two periods 1971-1990 and 1971-2008, with respective values of 47.5% and 46.5%. Assuming a model without fixed effects,

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[21] The GMM instruments are not observed before 1950.

[22] Baum et al. (2013) tested this effect for the euro area sample by introducing the years 2009-2010 and found a considerable upward effect on the threshold, especially in dynamic panel regression.

[23] The term fixed effects imply that although the intercept may differ across countries, it is time invariant. The fixed effects model allows for heterogeneity or individuality among countries.

[24] Testing for cross-sectional dependence is crucial for selecting the appropriate and efficient estimator. We use four tests: Lagrange multiplier (LM) test from Breusch and Pagan (1980), two tests of Pesaran (2004, 2006), one based on Lagrange multiplier and the other on pairwise correlation coefficients. The latter, has a lower power when the population average pair-wise correlations are zero (Pesaran et al., 2008). The fourth test is proposed by Pesaran and al. (2008), which developed a bias-adjusted test that is a modified version of the LM test.

[25] For comparison purpose with the regression kink, we also run a dynamic quadratic form ($g_t = c + \alpha d_{t-1} + \beta d_{t-1}^2 + \delta g_{t-1} + \varepsilon_t$) estimation for each country using GLS method over 1880-2010 (no observed instruments on this period) and GMM method for 1950-2010. Results, available upon request from the author, are not significant.

the estimates are statistically significant over three periods; 1971-1990, 1991-2008 and 1971-2008, with respectively 49.4%, 80.1% and 62.8% thresholds. The J statistics shows the efficiency of the instruments considered in the regressions.

Insert Table 4 about here

These results emphasized the drawbacks from which the GLS method suffers. It reveals also that the relationship is affected by high heterogeneity behaviors across countries. We deal with the first issue by continuing the estimation using the GMM method. For the heterogeneity issue, we considered size effect between countries and vary the sample removing sizable countries namely Japan, United Kingdom, and United States. We also present the results for the sample of the euro area countries as group of homogenous monetary and exchange rate regimes. Similarly, we filter countries according to a number of aggregates (government effectiveness and expenditure levels reflecting public economic governance).

For the fixed effects model (Table 5), the results of the estimates show a concave relationship between debt and growth for the period 1950-1970, 1971-1990 and 1971-2008. The results do not improve by excluding Japan alone. The two periods 1971-2008 and 1950-2008 on which the relationship becomes significant shows, a convexity of the relationship. Excluding Japan and the United States, the model improves over the period 1950-1970 and the threshold over this period is 78.4%. Adding the United Kingdom to the excluded countries, the results of the model remain substantially the same. The threshold varies from 40% over 1971-1990 to nearly 78% for the period 1950-1970. For the euro area, the relationship was concave and significant over the sub-periods 1950-1970, 1971-1990, 1991-2008 and 1971-2008. Estimates for the period 1950-2008 are rejected. The threshold ranges from a minimum of 45% over the period 1971-2008 to a maximum of 94% over the period 1991-2008.

Insert table 5 about here

We generate fixed effect for this model in Table A.4. It reports exactly the deviation of each country $(\gamma_i - \bar{\gamma})$ from the homogenous constant term (the average constant for the whole panel). These results emphasize high spreads across countries which makes it possible to conclude that the heterogeneity greatly affects the growth-debt linkages. This may explain the big differences in results across periods and samples. The threshold estimates are not tiny to consider as a claim for a one unique threshold for all countries in all times. On the contrary, this highly suggests that the growth debt relationship is rather country specific than a common one. Even when considering no fixed effects results (Table 6), no substantial improvement is found.

Insert Table 6 about here

For more investigation, we vary the samples by grouping countries according to their level of government effectiveness and expenditure levels reflecting good public economic governance. We split the sample into two groups of high and moderate level based on government effectiveness and shares to GDP of respectively total final government consumption, military expenditures and, government transfers and subsidies. The first variable is from Worldwide Governance Indicators of the World Bank, while other variables are from the World Development Indicators of World Bank database. Countries are filtered form high to low values on average over time and the frontier between high and moderate groups is determined according to the average of countries averages. Therefore, the average government consumption to GDP over 1960-2008 ranges from 10.1% for Switzerland to 23.6% recorded for Sweden. The average of the sample is 17.7% and this value is used to split the two groups. Similarly, the government effectiveness index average over 1996-2016 ranges from 0.53 for Italy to 2.09 for Finland, and its average over countries is 1.58 determine the moderate and high groups. The same approach was followed for the other aggregates.

Table A.5 summarizes the estimated thresholds by periods and country samples for the two groups of countries for each of the four considered variables. The estimation is done by GMM method. The results are fuzzy, and many non-significant thresholds are reported especially over the period 1951-2008. However, the recent period seems to point out to more homogeneity in countries behavior and significant thresholds, though different by sample, are reported. These results emphasize the idea of studying the debt-growth relationship on country by country case.

The investigation (both preliminary data analysis and estimation) revealed high heterogeneity in data and behaviors across countries. Despite that we could sometimes prove the existence of a debt threshold by advanced econometric methods, the heterogeneity suggests that this one tend to be rather country specific than common rule for all countries. Furthermore, when dealing with panel cross-section data, it is usually assumed that cross section errors in panel data models are independent, especially for large cross-section size. However, the presence of such cross-section dependence in estimation can result in serious problems of efficiency loss and inappropriate statistical tests.

7. Conclusion

In this paper, we studied the existence of a possible threshold effects in the relationship between public debt and economic growth. We used two econometric approaches. The first method, applied individually to each country of the sample of 20 advanced countries, is the kink regression method developed by Hansen (2017), which searches endogenously for an unknown threshold. The second method is a technique that previously was explored by some authors especially Checherita and Rother (2010), which we applied to the panel of the previous countries.

Both methods argue clearly against the claim of a common debt threshold that fit all countries. It reveals that the threshold, whenever it exists, is rather a country specific than a common rule to fit all. Unlike the whole empirical literature examining the existence and values of debt thresholds on a cross-section data, our analysis undertakes the question on both cross-section and case by case examination over a long data span. Country-specific analysis highlighted, in fact, diverse types of relationship between growth and public debt. Accordingly, some countries can grow with high debt to GDP ratios; others could see their growth shrink from even low debt ratios, while growth in some others is insensitive to public debt. The study reveals also the instability of the relationship over time; almost every country exhibits different relationship by period, especially when the transition is between periods known for some specific changes in the international economic and monetary system.

The results drawn from our study point to further interesting developments since several economic and institutional variables such as interest rate and governance could be integrated into the analysis as they could have some notable effects on the debt-growth nexus beyond the simple model developed here.

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Tables and Figures

Table 1: Empirical studies survey of the public debt and economic growth threshold

	Authors	Samples	Econometric Methodologies	Debt thresholds	Other notes on methods and results
1	Reinhart and Rogoff (2010)	20 developed countries and 24 developing countries, 1946-2009	Growth mean and the median analysis according to pre-established thresholds	90% for advanced countries	Arbitrarily set thresholds: 30%, 60% and 90% and conclude that growth is altered from 90%.
2	Caner et al. (2010)	26 developed and 75 developing countries: 1980-2008	Hansen (2000) OLS threshold econometric estimate	77% and 64%	77% for the whole sample, 64% for developing countries.
3	Herndon et al. (2013)	The 20 developed countries of RR (2010)	Replication of RR (2010) work after correction of data errors	No threshold found	The relationship is sensitive to country and period
4	Kumar and Woo (2010)	38 countries, most of which are OECD countries: 1970-2007	Econometric estimation by various methods such as GMM	High level but not reported	Correction of problems of reverse causality, endogeneity and heterogeneity
5	Checherita and Rother (2010)	12 euro area countries: 1970-2009	Estimation of a polynomial form of growth explained by debt ratio	Interval [70% -100%]	The degree of the polynomial form is around 1.2 to 3
6	Checherita et al. (2014)	11 Euro, 22 OECD and 14 EU countries: 1960-2010	Cobb-Douglas optimization augmented by public spending	65%: OECD; 63%: the EU; 50%: euro	The debt threshold is a non-linear function of the elasticity of public expenditure/private capital ratio
7	Pescatori et al. (2014)	34 mainly developed countries: 1875-2011	Analysis of growth(t + k) and debt(t); k={1,5,10,15} by a descriptive approach	No threshold found	High government debt tends to increase economic growth volatility.
8	Chang and Chiang (2009)	15 OECD countries: 1990-2004	Threshold method for non-dynamic panel	32.3% and 66.25%	The relationship is positive over the three regimes delimited by the two thresholds
9	Cecchetti et al. (2011)	18 OECD countries: 1980-2010	Threshold method for non-dynamic panel	85%	Debt negatively influences growth above 85% threshold.
10	Minea and Parent (2012)	20 developed countries of RR (2010) and Abbas et al. (2010) data	Estimation of econometric relationship with changing thresholds	60%, 90% and 115%	Between 90% and 115%, negative effect, between 60% and 90% and above 115% positive.
11	Egert (2015)	The RR (2010) data and countries sample	Estimation and detection of endogenous thresholds (Hansen, 1999)	20%: Central debt; 50%: General debt	Individual estimate confirms threshold for some countries around 30% (United States)
12	Baglan and Yoldas (2013)	20 developed countries of RR (2010): 1954-2008	Inference techniques to remedy to endogeneity and heterogeneity issues	Between 18% and 53%	Non-robust threshold and subject to uncertainty
13	Baum et al. (2013)	12 euro countries: 1990-2010	Threshold method for dynamic and non-dynamic panel	67%	Study of short-term impact
14	Kourtellos et al. (2012)	Sample of 82 countries: 1980-2009	Threshold regressions using the Solow growth model	No threshold found	Heterogeneity and influence of institutional quality

Table 1 (continued)

	Authors	Samples	Econometric Methodologies	Debt thresholds	Other notes on methods and results
15	Eberhardt and Presbitero (2015)	Sample of 105 countries: 1970-2008	Techniques addressing heterogeneity and dependence in cross-sections	No common threshold	Estimation of dynamic and static non-linear models by GMM method
16	Panizza and Presbitero (2012)	17 OECD countries: 1981-2008	Estimation by GMM of linear and non-linear relationships	No threshold found	No negative effect of the debt on growth
17	Sharpe (2013)	12 euro countries: 1998-2011	Estimated relationship of debt and interest rate by GMM and TSLS	40% and 133%	Negative effect for debt ratios over 40%, emphasized above 133% threshold
18	Pan and Wang (2013)	12 euro countries: 1970-2009	Bayesian analysis using dynamic factor models	Negative relationship	Common factors and shocks affect positive growth and negative debt
19	Gomez-Puig and Sosvilla-Rivero (2015)	11 euro countries: 1980-2013	Granger (1969) causality method	56% to 103%	Causality dependent on the country, the threshold found only for 4 countries
20	Di Sanzo and Bella (2015)	12 countries of the euro; 1970-2012	Studies of individual causality by nonparametric tests	Threshold not examined	Causality results vary across countries
21	Greiner (2011)	Long-term simulation for Italy, Germany and the euro zone	Simulated endogenous growth model	Threshold not examined	The impact of debt on Growth is positive if the pace of debt remains lower than the pace of GDP
22	Marchionne and Parekh (2015)	Sample of 27 countries: 1994-2010	Estimation considering the GINI index	Non-linearity; No threshold reported	The results suggest non-linear link that depends on the income distribution
23	Lin (2014)	62 developing and developed countries: 1991-2005	Threshold quantile Lasso regression	Thresholds ranging from 10% to 100%	Thresholds vary by country and quintile and more common in developing than in developed countries
24	Lee et al. (2017)	RR (2010) database for 20 developed countries	Test for threshold effects by regressing growth median on public debt	Around 30%	The median real GDP growth falls abruptly above a debt to GDP ratio of 30%
25	Gomez-Puig and Sosvilla-Rivero (2017)	11 Euro area countries; 1961-2015	Time series regressions based on economic growth literature	Variable threshold from 21% to 61%	Threshold varies across countries from 21% in France to 61% in Belgium
26	Chudik (2017)	40 advanced and developing countries: 1965-2010	Test for thresholds in dynamic heterogeneous panel with cross-sectionally dependent errors	No evidence of any threshold	Significant negative long-run effects of public debt build-up on output growth
27	Syssoyeva-Masson and De Sousa Andrade (2017)	60 developed and developing countries, IMF data: 1970-2012	Panel regression using time series cointegration and Hansen (1999)	Two regimes 44% and 48%	Highlight the debt long memory process and recommend the long run analysis

Table 2: Regression kink results over long periods

Specification	$g_t = \beta_1(d_{t-1} - \gamma)_- + \beta_2(d_{t-1} - \gamma)_+ + \delta g_{t-1} + c + e_t$					
Country	β1	β2	δ	c	γ	Period
Australia	0.002 (0.321)	1.719 (0.234)	0.49 (0.503)	1.11 (2.996)	134 (167.9)	1901-2010
Austria	0.070 (0.072)	-0.04 (0.039)	0.584 (0.085)	2.389 (1.335)	35.6 (6.4)	1881-2010
Belgium	0.000 (0.000)	0.088 (0.394)	0.214 (0.135)	1.487 (0.333)	173.4 (231.3)	1881-2010
Canada	1.016 (1.133)	0.002 (0.016)	0.314 (0.127)	1.471 (0.978)	22.7 (4.1)	1881-2010
Denmark	-0.067 (0.058)	0.048 (0.04)	0.016 (0.174)	0.679 (0.793)	33.4 (6.8)	1881-2010
France	-0.029 (0.01)	0.167 (0.064)	-0.203 (0.147)	-0.306 (1.22)	161 (20.4)	1881-2010
Germany	0.318 (0.127)	-0.051 (0.033)	0.459 (0.148)	2.587 (0.904)	20.1 (2.3)	1881-2010
Greece	-0.008 (0.016)	-2.063 (0.194)	0.027 (0.07)	1.172 (2.576)	215 (1.5)	1884-2010
Ireland	0.177 (0.191)	0.04 (0.016)	0.435 (0.15)	1.07 (0.829)	35.1 (7.9)	1929-2010
Italy	-0.058 (0.035)	0.053 (0.053)	0.371 (0.146)	0.001 (1.051)	66 (18.8)	1881-2010
Japan	-0.434 (0.159)	-0.006 (0.008)	-0.11 (0.104)	2.564 (0.671)	22.8 (4.9)	1881-2010
Netherlands	0.04 (1.806)	-0.009 (1.182)	0.233 (3.205)	3.235 (26.27)	110.9 (2439)	1881-2010
New Zealand	-0.005 (0.01)	0.074 (0.112)	-0.048 (0.107)	0.814 (0.973)	163.3 (78.2)	1881-2010
Norway	2.647 (0.87)	0.032 (0.028)	-0.123 (0.14)	2.501 (0.705)	14.6 (1.4)	1881-2010
Portugal	125.31 (17.9)	-0.069 (0.022)	-0.106 (0.137)	4.482 (0.954)	13.6 (0.0)	1881-2010
Spain	-0.059 (0.018)	0.017 (0.028)	0.014 (0.098)	0.399 (0.796)	74.6 (16.2)	1881-2010
Sweden	6.189 (0.01)	-0.001 (0.048)	0.109 (0.102)	2.092 (1.21)	13.4 (16.3)	1881-2010
Switzerland	-0.023 (0.016)	3.977 (0.595)	0.047 (0.119)	0.272 (0.63)	74.3 (0.6)	1899-2010
United Kingdom	-30.542 (0.02)	0.00 (0.235)	0.362 (0.067)	0.932 (1.571)	27.5 (55.0)	1881-2010
United States	34.306 (8.17)	-0.013 (0.02)	0.292 (0.107)	3.082 (0.936)	7.6 (0.1)	1881-2010

Note: Standard errors are given in parentheses.

Table 3: Regression kink results over the period 1956-2010

Specification	$g_t = \beta_1(d_{t-1} - \gamma)_- + \beta_2(d_{t-1} - \gamma)_+ + \delta g_{t-1} + c + e_t$				
Country	β1	B2	δ	c	γ
Australia	0.034 (0.033)	0.614 (0.155)	0.119 (0.167)	2.259 (0.697)	37.4 (1.1)
Austria	-0.086 (0.038)	0.039 (0.025)	-0.017 (0.141)	1.555 (0.517)	44.0 (7.7)
Belgium	51.31 (4.898)	-0.016 (0.024)	0.157 (0.107)	2.923 (0.694)	38.9 (3.8)
Canada	0.149 (0.238)	-0.006 (0.019)	0.328 (0.138)	1.587 (0.634)	52.7 (10.9)
Denmark	-36.619 (0.028)	-0.007 (0.095)	0.033 (0.024)	2.390 (1.191)	4.4 (15.4)
France	0.521 (0.478)	-0.024 (0.014)	0.476 (0.16)	1.705 (0.667)	16.1 (1.3)
Germany	-0.132 (0.061)	0.005 (0.033)	0.038 (0.183)	1.326 (0.817)	34.8 (7.7)
Greece	-0.058 (0.035)	0.053 (0.053)	0.371 (0.146)	0.001 (1.051)	66 (18.8)
Ireland	3.28 (1.227)	0.037 (0.015)	0.346 (0.128)	1.456 (0.667)	27.3 (1.0)
Italy	-0.053 (0.014)	0.091 (0.045)	0.100 (0.197)	0.502 (0.438)	105.4 (4.7)
Japan	-0.339 (2.241)	-0.011 (0.016)	0.299 (0.2)	2.171 (2.383)	18.1 (75.7)
Netherlands	0.083 (0.039)	-0.064 (0.113)	0.162 (0.179)	2.821 (0.962)	61.7 (10.2)
New Zealand	1.81 (0.055)	-0.014 (0.023)	-0.074 (0.169)	2.058 (0.771)	19.4 (0.4)
Norway	-0.037 (0.023)	0.793 (0.007)	0.337 (0.139)	0.926 (0.677)	57.9 (0.7)
Portugal	10.74 (2.203)	-0.066 (0.021)	0.377 (0.132)	3.998 (0.95)	14.5 (0.2)
Spain	-0.064 (0.078)	0.038 (0.026)	0.549 (0.126)	0.543 (0.841)	34.5 (18.7)
Sweden	-0.119 (0.044)	0.056 (0.025)	0.394 (0.156)	-0.198 (0.579)	40.2 (5.6)
Switzerland	-0.083 (0.036)	0.05 (0.037)	0.177 (0.144)	0.288 (0.492)	39.9 (8.8)
United Kingdom	0.124 (0.072)	-0.021 (0.019)	0.154 (0.18)	3.037 (0.873)	59 (8.5)
United States	1.892 (0.617)	-0.072 (0.028)	0.028 (0.156)	4.650 (0.800)	34.4 (0.7)

Note: Standard errors are given in parentheses.

Table 4: GLS and GMM panel estimation results

Regression by Generalized Least Squares method (GLS). Specification: $g_{i,t+5} = \alpha d_{i,t} + \beta d_{i,t}^2 + \gamma_i + \varepsilon_{i,t}$										
Model with fixed effects										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	0.010	*0.647*	-0.017	*0.309*	0.049**	*0.030*	-0.003	*0.770*	-0.045***	*0.000*
β	0.000	*0.689*	0.000	*0.789*	0.000	*0.196*	0.000	*0.801*	0.000***	*0.000*
γ	3.296***	*0.000*	2.685***	*0.000*	-0.470	*0.620*	2.229***	*0.000*	4.206***	*0.000*
Threshold	NS		NS		NS		NS		123.3[a]	
Model without fixed effects										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	-0.038***	*0.000*	-0.014	*0.307*	0.015	*0.234*	-0.005	*0.576*	-0.038***	*0.000*
β	0.000***	*0.007*	0.000	*0.546*	0.000	*0.251*	0.000	*0.841*	0.000***	*0.000*
γ	4.619***	*0.000*	2.411***	*0.000*	1.448***	*0.002*	2.165***	*0.000*	3.802***	*0.000*
Threshold	139.0[a]		NS		NS		NS		107.0[a]	
Regression by Generalized Method of Moments (GMM). Specification: $g_{i,t+5} = \alpha d_{i,t} + \beta d_{i,t}^2 + \varphi Inst + \gamma_i + \varepsilon_{i,t}$										
Model with fixed effects										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	0.159	*0.671*	0.190**	*0.014*	-0.204	*0.592*	0.093**	*0.013*	0.060	*0.469*
β	-0.001	*0.733*	-0.002***	*0.006*	0.002	*0.389*	-0.001***	*0.004*	-0.001	*0.169*
γ	-0.444	*0.957*	-0.707	*0.625*	5.571	*0.690*	-0.014	*0.989*	3.290**	*0.019*
J-statistic[b]	2.755	*0.097*	3.923	*0.141*	10.774	*0.001*	0.334	*0.563*	3.692	*0.055*
Threshold	NS		47.5		NS		46.5		NS	
Model without fixed effects										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	0.038	*0.808*	0.234*	*0.069*	0.161**	*0.043*	0.106**	*0.016*	0.566	*0.186*
β	-0.001	*0.529*	-0.002*	*0.081*	-0.001**	*0.034*	-0.001***	*0.005*	-0.006	*0.157*
γ	4.176*	*0.099*	-2.845	*0.277*	-3.684	*0.202*	-0.682	*0.616*	-6.376	*0.397*
J-statistic[b]	4.526	*0.339*	2.949	*0.399*	0.786	*0.675*	1.143	*0.565*	0.666	*0.717*
Threshold	NS		49.4		80.1		62.8		NS	

Notes: Significant at 1% (***), 5% (**) and 10% (*). NS is Non-Significant.
a: Coefficients are statistically significant, but the function is convex which means that the debt affects negatively growth for debt ratios below this threshold and positively above this threshold.
b: J-statistic is the value of the GMM objective function.
The threshold is calculated according to the formulae: $\hat{d} = -\frac{\alpha}{2\beta}$.

Table 5: GMM results for varying countries sample (fixed effects model)

Sample of 20 OECD countries										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	0.159	*0.671*	0.190**	*0.014*	-0.204	*0.592*	0.093**	*0.013*	0.060	*0.469*
β	-0.001	*0.733*	-0.002***	*0.006*	0.002	*0.389*	-0.001***	*0.004*	-0.001	*0.169*
γ	-0.444	*0.957*	-0.707	*0.625*	5.571	*0.690*	-0.014	*0.989*	3.290**	*0.019*
J-statistic[b]	2.755	*0.097*	3.923	*0.141*	10.774	*0.001*	0.334	*0.563*	3.692	*0.055*
Threshold	NS		47.5		NS		46.5		NS	
Sample of 19 OECD countries (Japan excluded)										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	0.137	*0.457*	0.184**	*0.015*	-0.981***	*0.007*	0.098**	*0.019*	-0.172***	*0.000*
β	-0.001	*0.579*	-0.002***	*0.008*	0.009***	*0.005*	-0.001***	*0.008*	0.001**	*0.013*
γ	-0.364	*0.932*	-0.764	*0.597*	23.638***	*0.007*	0.113	*0.912*	7.298***	*0.000*
J-statistic[b]	2.646	*0.104*	3.097	*0.213*	6.874	*0.009*	0.149	*0.700*	4.641	*0.098*
Threshold	NS		40.8		56.6[a]		51.5		75.5[a]	
Sample of 18 OECD countries (Japan and USA excluded)										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	0.344**	*0.017*	0.186**	*0.015*	-0.731**	*0.018*	0.091**	*0.024*	-0.162***	*0.000*
β	-0.002**	*0.041*	-0.002***	*0.008*	0.007**	*0.012*	-0.001***	*0.010*	0.001**	*0.011*
γ	-4.910	*0.138*	-0.731	*0.609*	16.307**	*0.028*	0.296	*0.766*	7.139***	*0.000*
J-statistic[b]	5.375	*0.146*	2.402	*0.301*	11.895	*0.003*	0.372	*0.542*	3.692	*0.055*
Threshold	78.4		40.8		54.0[a]		51.0		79.3[a]	
Sample of 17 OECD countries (Japan, USA and UK excluded)										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	0.296**	*0.033*	0.162**	*0.028*	-0.500*	*0.052*	0.070**	*0.047*	-0.172**	*0.011*
β	-0.002*	*0.074*	-0.002**	*0.013*	0.005**	*0.030*	-0.001**	*0.017*	0.001*	*0.075*
γ	-3.428	*0.269*	-0.256	*0.851*	10.658*	*0.088*	0.832	*0.333*	7.496***	*0.000*
J-statistic[b]	3.912	*0.418*	1.916	*0.384*	17.670	*0.000*	0.389	*0.823*	7.681	*0.021*
Threshold	76.5		40.1		51.9[a]		48.6		80.3[a]	
Sample of 10 Euro countries										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*	Coef.	*prob.*
α	1.929*	*0.096*	0.100*	*0.057*	1.063**	*0.013*	0.064*	*0.071*	-0.123**	*0.019*
β	-0.014*	*0.095*	-0.001**	*0.027*	-0.006*	*0.092*	-0.001**	*0.021*	0.001	*0.126*
γ	-37.593	*0.136*	0.655	*0.557*	-41.709***	*0.006*	1.443*	*0.085*	6.689***	*0.000*
J-statistic[b]	2.191	*0.534*	3.652	*0.161*	4.996	*0.172*	0.765	*0.382*	4.387	*0.112*
Threshold	69.6		46.1		94.0		45.0		NS	

Notes: Significant at 1% (***), 5% (**) and 10% (*). NS is Non-Significant.

a: Coefficients are statistically significant, but the function is convex which means that the debt affects negatively growth for debt ratios below this threshold and positively above this threshold.

b: J-statistic is the value of the GMM objective function.

The threshold is calculated according to the formulae: $\hat{d} = -\frac{\alpha}{2\beta}$.

Table 6: GMM estimation results for reduced samples for model without fixed effects

Sample of 20 OECD countries										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*
α	0.038	*0.808*	0.234*	*0.069*	0.161**	*0.043*	0.106**	*0.016*	0.566	*0.186*
β	-0.001	*0.529*	-0.002*	*0.081*	-0.001**	*0.034*	-0.001***	*0.005*	-0.006	*0.157*
γ	4.176*	*0.099*	-2.845	*0.277*	-3.684	*0.202*	-0.682	*0.616*	-6.376	*0.397*
J-statistic^b	4.526	*0.339*	2.949	*0.399*	0.786	*0.675*	1.143	*0.565*	0.666	*0.717*
Threshold	NS		49.4		80.1		62.8		NS	
Sample of 19 OECD countries (Japan excluded)										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*
α	0.054	*0.753*	0.291**	*0.037*	0.172*	*0.092*	0.136***	*0.003*	0.711	*0.357*
β	-0.001	*0.526*	-0.003**	*0.042*	-0.001*	*0.088*	-0.001***	*0.001*	-0.008	*0.322*
γ	3.657	*0.210*	-3.871	*0.166*	-3.976	*0.280*	-1.394	*0.304*	-9.868	*0.508*
J-statistic^b	6.446	*0.168*	0.764	*0.858*	4.645	*0.326*	0.571	*0.752*	3.765	*0.152*
Threshold	NS		47.5		86.0		60.7		NS	
Sample of 18 OECD countries (Japan and United States excluded)										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*
α	0.033	*0.799*	0.254**	*0.036*	0.162*	*0.074*	0.109***	*0.002*	0.270	*0.416*
β	-0.001	*0.502*	-0.003**	*0.039*	-0.001*	*0.063*	-0.001***	*0.001*	-0.004	*0.339*
γ	4.113**	*0.049*	-2.873	*0.220*	-3.160	*0.310*	-0.524	*0.592*	-0.656	*0.904*
J-statistic^b	5.507	*0.239*	0.744	*0.863*	4.562	*0.335*	0.242	*0.886*	5.574	*0.062*
Threshold	NS		46.0		72.8		57.4		NS	
Sample of 17 OECD countries (Japan, United States and United Kingdom excluded)										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*
α	0.247*	*0.096*	0.223*	*0.066*	0.415**	*0.021*	0.109***	*0.003*	0.249*	*0.069*
β	-0.003*	*0.055*	-0.002*	*0.063*	-0.003*	*0.051*	-0.001***	*0.001*	-0.003**	*0.045*
γ	0.104	*0.966*	-2.088	*0.364*	-12.292**	*0.021*	-0.601	*0.569*	-1.585	*0.541*
J-statistic^b	2.873	*0.412*	0.386	*0.943*	0.319	*0.853*	0.594	*0.743*	7.465	*0.058*
Threshold	36.55		45.31		76.62		59.12		45.09	
Sample of 10 Euro zone countries										
	1950-1970		1971-1990		1991-2008		1971-2008		1950-2008	
	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*	coef.	*prob.*
α	0.285*	*0.101*	0.226*	*0.061*	0.449**	*0.035*	0.074*	*0.065*	0.173	*0.214*
β	-0.004*	*0.091*	-0.002*	*0.053*	-0.002**	*0.041*	-0.001**	*0.041*	-0.003*	*0.098*
γ	0.148	*0.957*	-2.624	*0.339*	-16.135*	*0.057*	0.591	*0.544*	1.741	*0.423*
J-statistic^b	4.306	*0.23*	0.971	*0.615*	0.843	*0.656*	2.066	*0.559*	12.577	*0.002*
Threshold	36.6		55.4		91.7		52.6		NS	

Notes: significant at 1% (***), 5% (**) and 10% (*). NS is Non-Significant.

b: J-statistic is the value of the GMM objective function.

The threshold is calculated according to the formulae: $\hat{d} = -\frac{\alpha}{2\beta}$.

Figure 1: Regression kink results for United States, Japan, United Kingdom and Australia

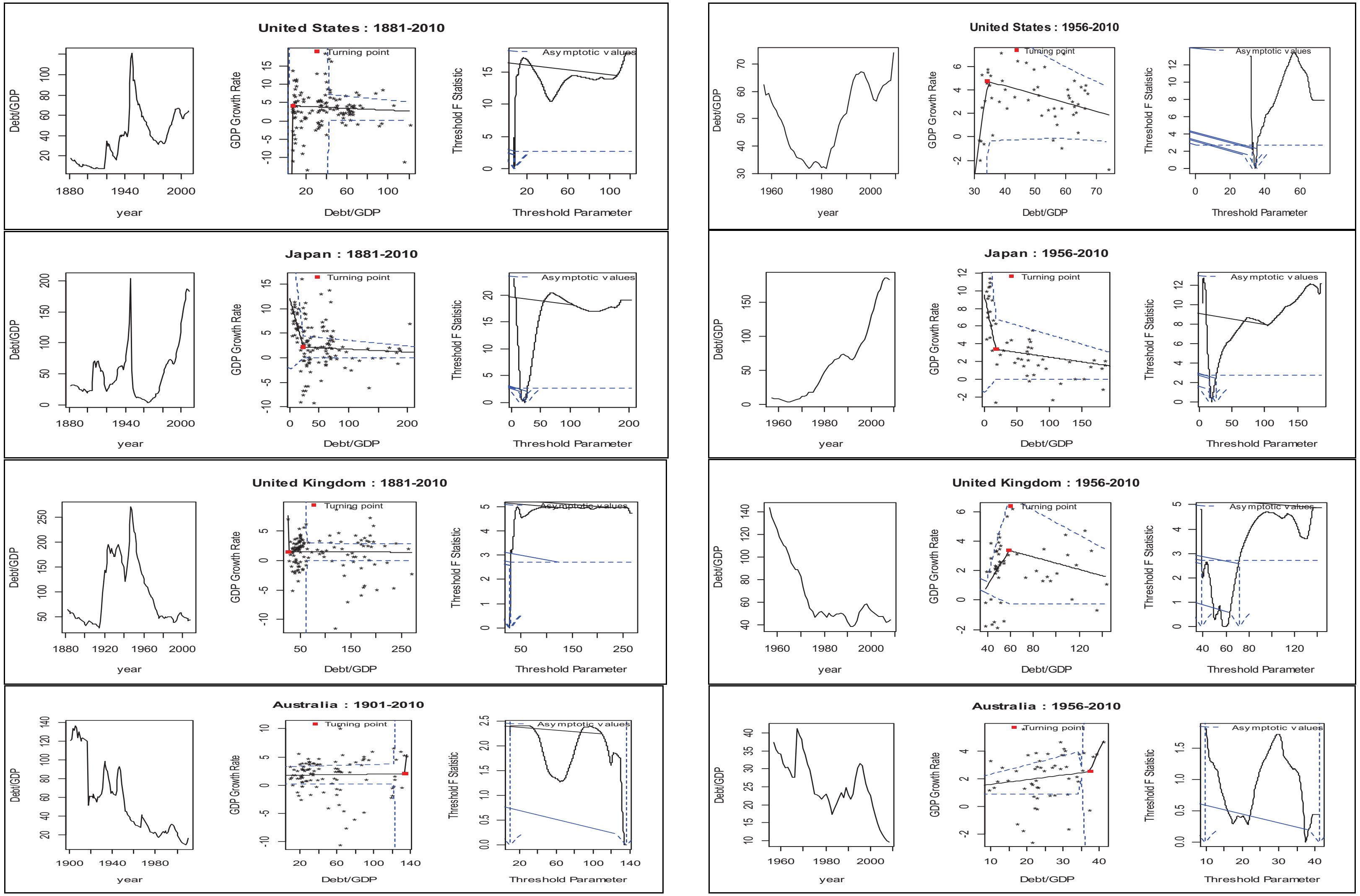


Figure 2: Regression kink results for Austria, Belgium, Canada and Denmark

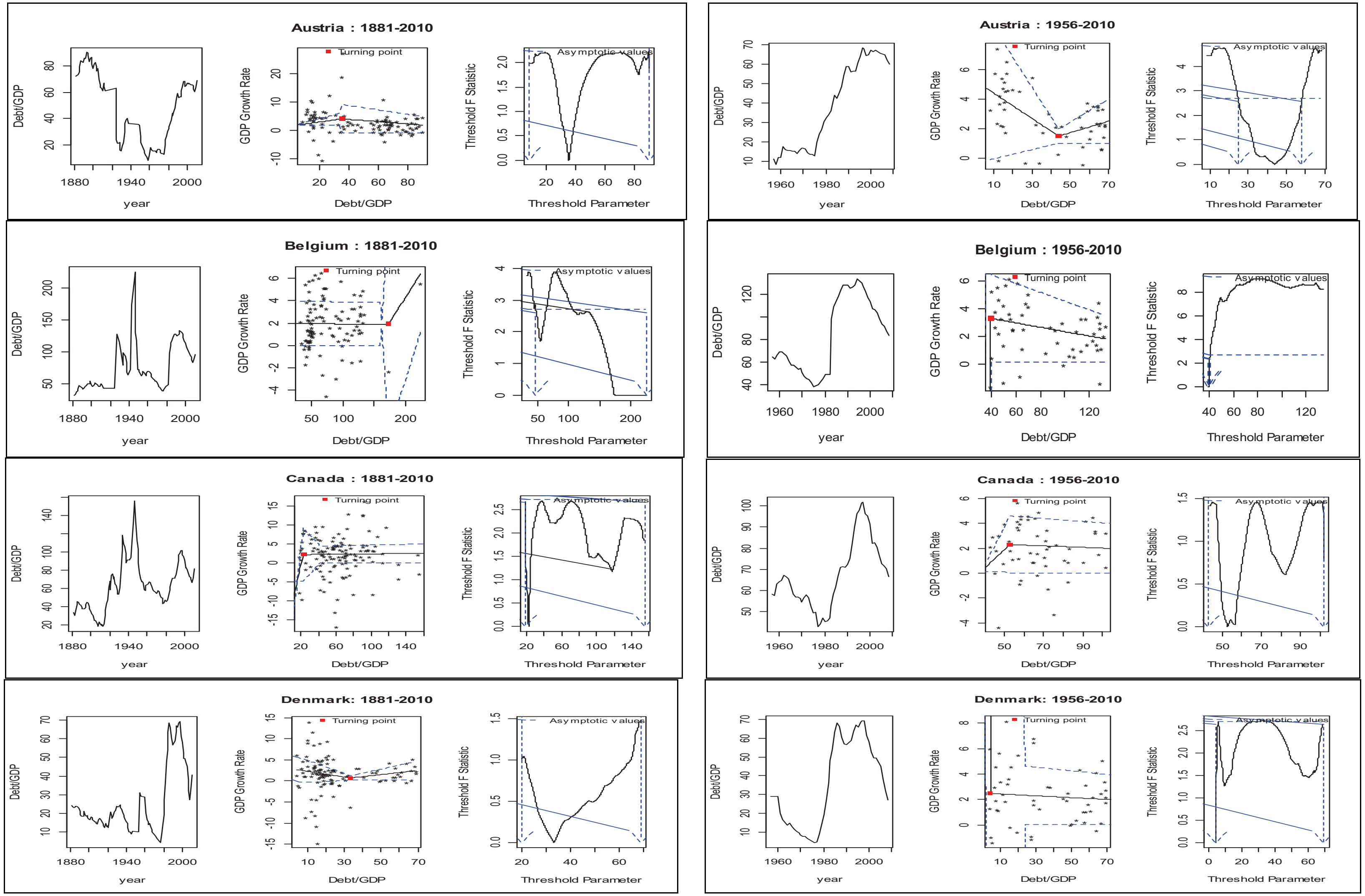


Figure 3: Regression kink results for France, Germany, Italy and Ireland

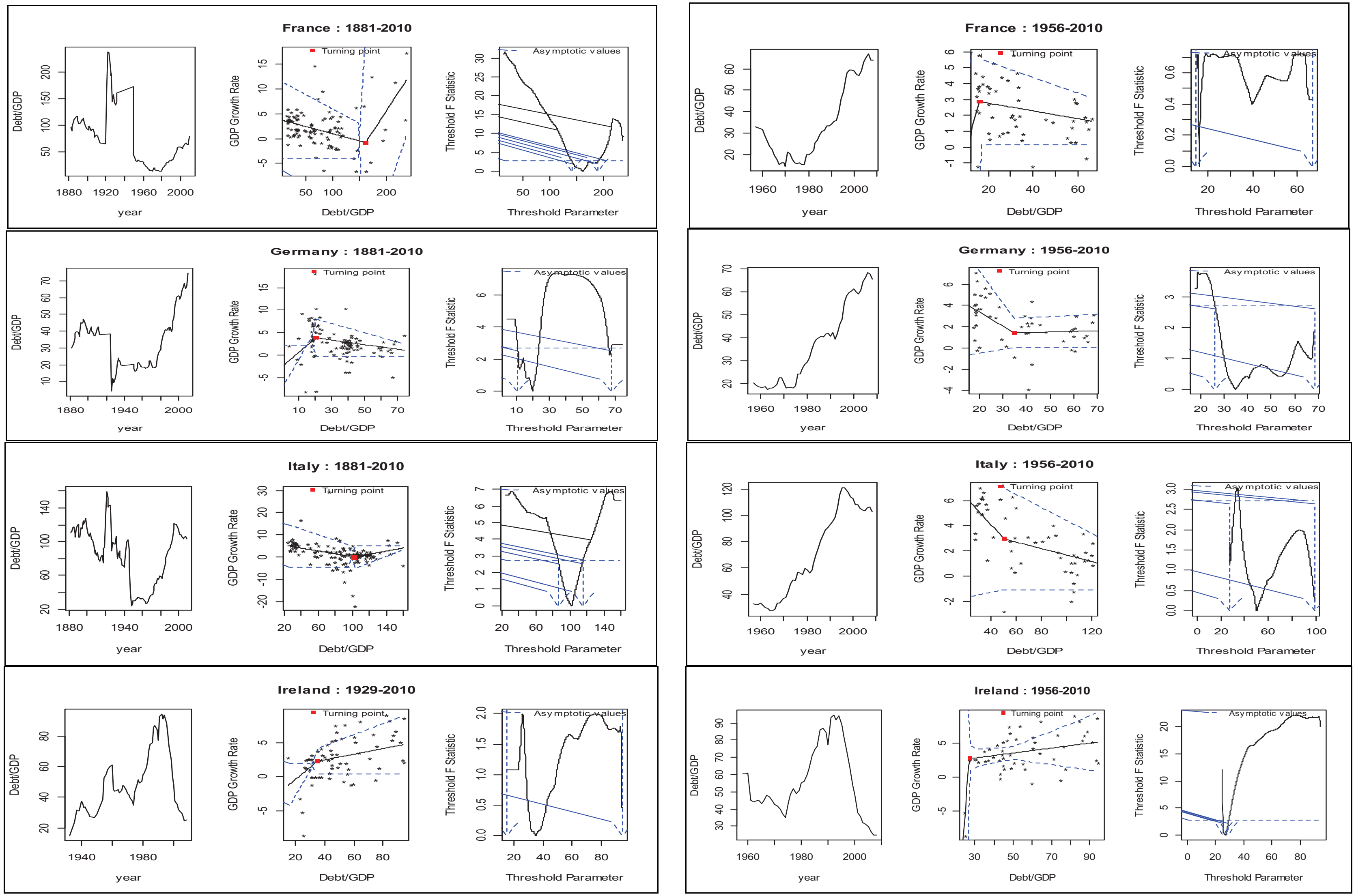


Figure 4: Regression kink results for Portugal, Spain, Sweden and Switzerland

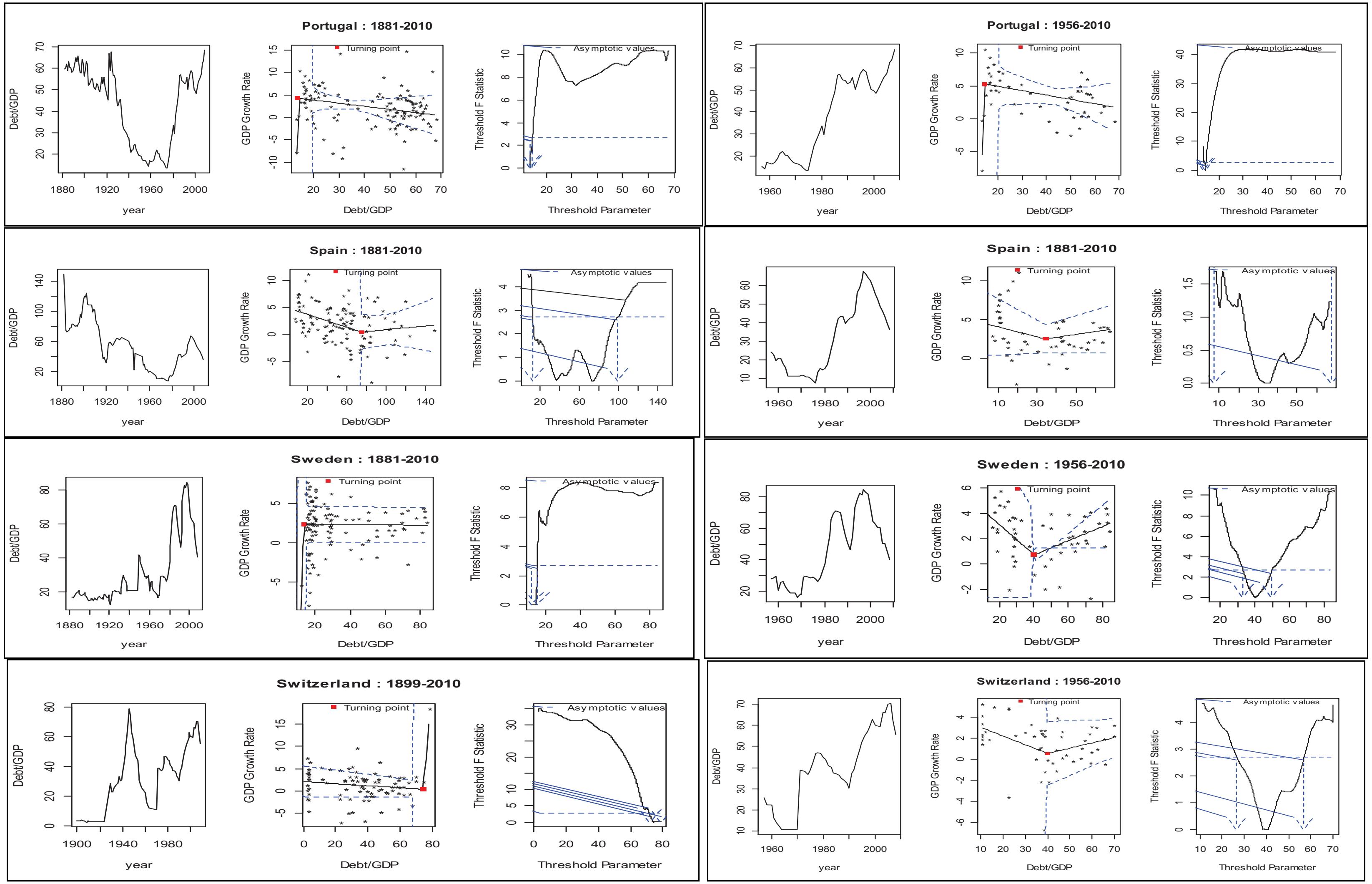


Figure 5: Regression kink results for New Zealand, Greece, Netherlands and Norway

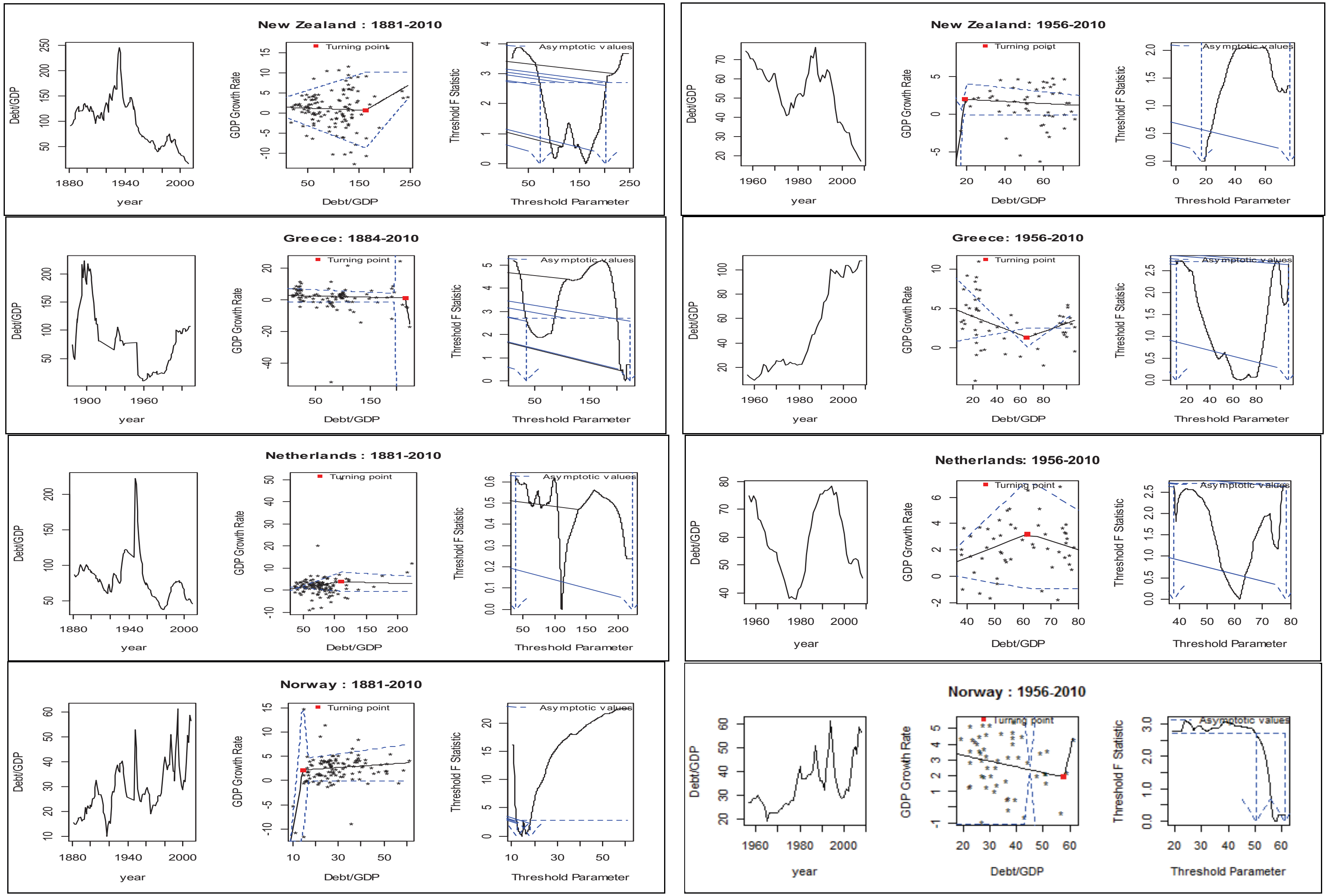


Figure 6: 5-year lead economic growth and public debt over different periods (panel of 20 advanced countries)

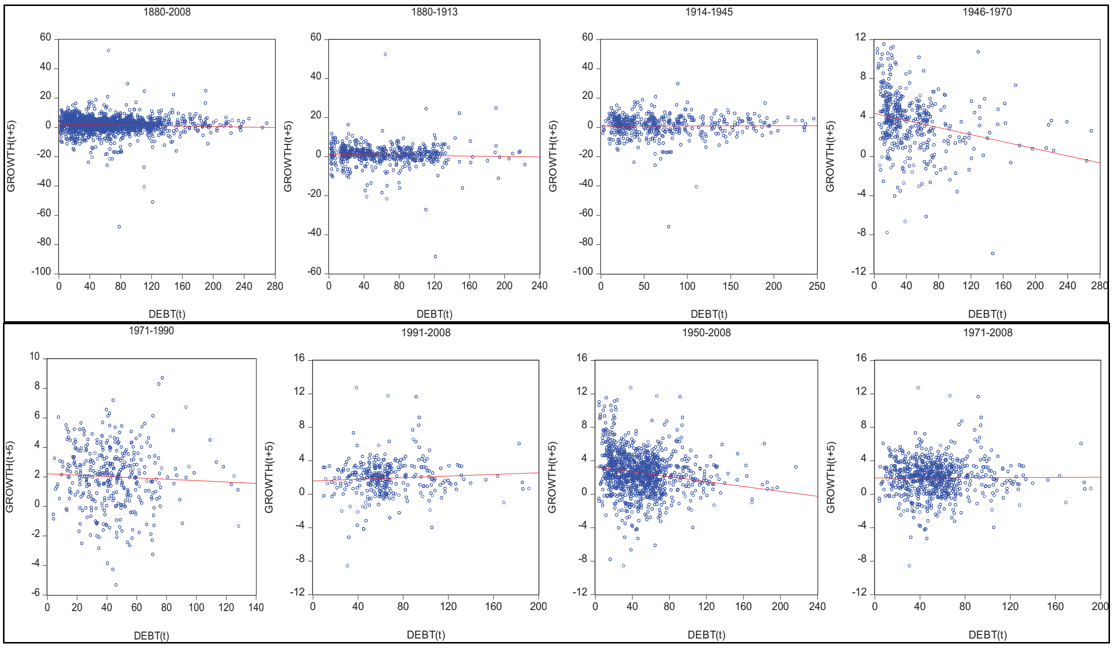
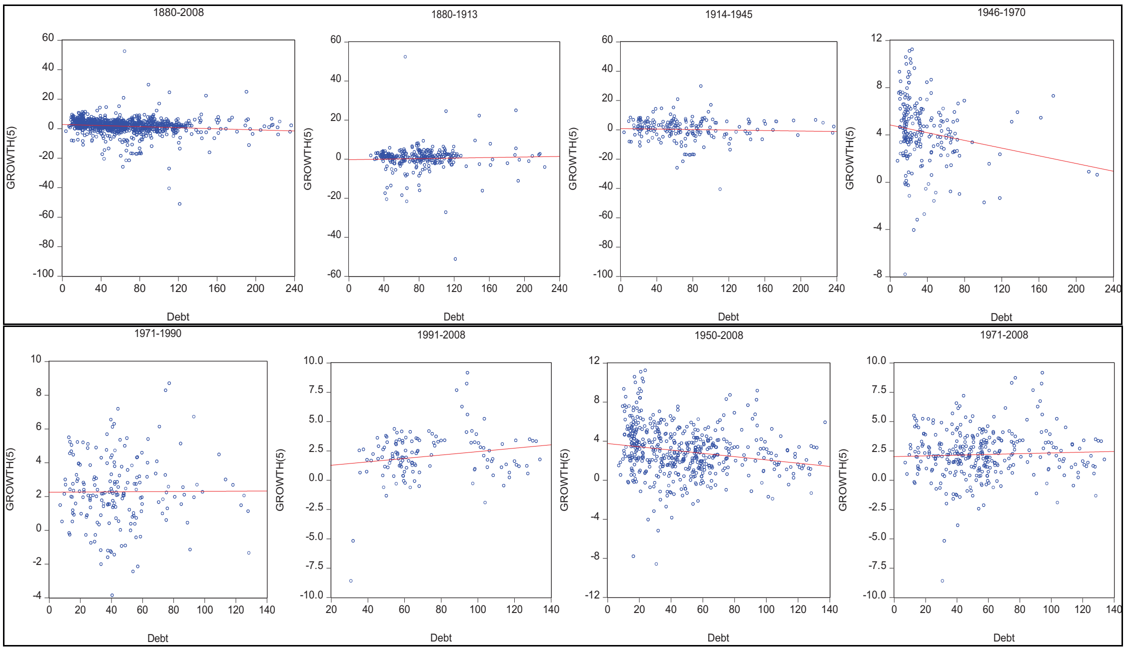


Figure 7: 5-year lead economic growth and public debt over different periods (panel of 10 euro countries)



Appendix A: Tables

Table A.1. Public debt and economic growth correlations by countries and periods

	Correlation of debt(t) and growth(t)					Correlation of debt(t) and growth(t+5)				
Countries	1950-2008	1950-1970	1971-1990	1991-2008	1971-2008	1950-2008	1950-1970	1971-1990	1991-2008	1971-2008
Australia	0.08	-0.25	0.26	0.14	0.13	-0.07	-0.11	-0.07	-0.44	-0.22
Austria	-0.54***	0.20	-0.28	0.15	-0.3	-0.51***	0.12	-0.52	0.39	-0.38**
Belgium	-0.40**	-0.30	-0.21	-0.19	-0.28*	-0.47***	-0.12	-0.72**	-0.17	-0.44**
Canada	-0.1	-0.02	-0.07	0.14	-0.10	-0.37**	-0.27	-0.34	-0.61	-0.44**
Denmark	-0.13	-0.26	0.08	0.25	0.08	-0.28**	-0.18	-0.06	0.27	-0.06
France	-0.44***	-0.25	-0.12	0.21	-0.28*	-0.54***	-0.17	-0.49	0.31	-0.25
Germany	-0.51***	-0.26	-0.29	-0.13	-0.19	-0.57***	0.00	-0.21	-0.10	-0.30
Greece	-0.30**	0.30	-0.14	0.15	0.04	-0.43**	0.01	-0.11	0.72**	0.17
Ireland	0.27**	-0.22	0.13	0.41*	0.32**	-0.22	0.16	-0.51	-0.31	-0.36*
Italy	-0.70***	0.09	0.00	0.25	-0.38**	-0.66***	-0.22	-0.17	-0.16	-0.28
Japan	-0.68***	-0.11	0.06	-0.03	-0.44**	-0.75***	-0.05	-0.23	-0.24	-0.55**
Netherlands	0.06	-0.23	0.16	0.00	0.12	-0.21	-0.1	-0.41	-0.23	-0.21
New Zealand	0.11	0.22	-0.05	0.06	-0.08	-0.02	0.07	0.02	-0.40	-0.25
Norway	-0.25*	-0.18	-0.22	-0.19	-0.30	-0.47***	-0.22	-0.69**	-0.74**	-0.74***
Portugal	-0.35**	0.21	-0.04	-0.28	-0.23	-0.32**	0.18	-0.26	-0.24	-0.21
Spain	-0.29**	-0.36	-0.14	0.54**	-0.16	-0.50***	-0.3	-0.85**	-0.26	-0.36*
Sweden	-0.23*	0.04	0.16	0.01	0.11	-0.52***	-0.23	-0.28	-0.67*	-0.43**
Switzerland	-0.28**	0.37*	-0.23	0.58**	0.04	-0.19	0.21	-0.03	0.73**	-0.02
United Kingdom	-0.04	-0.20	0.24	0.55**	0.36**	-0.14	-0.3	-0.09	-0.58	-0.42
United States	0.01	0.11	0.06	-0.17	-0.09	-0.1	0.03	-0.18	-0.35	-0.21
Total Sample	-0.25*	-0.34	-0.02	-0.04	-0.07	-0.29**	-0.24	-0.14	-0.23	-0.11
Significant Correlations	13/20	1/20	0/20	4/20	6/20	13/20	0/20	3/20	4/20	8/20
Negatives correlations	15/20	12/20	11/20	6/20	12/20	20/20	12/20	19/20	16/20	19/20

Note: Significant at 1% (***), 5% (**) and 10% (*).

Table A.2. Data heterogeneity tests across sections and periods

Test for Equality of **means, medians and variances** of GROWTH Categorized by values of DEBT

Equality of **means**	Sample: 1881 2008			Sample: 1950 2008			Sample: 1971 2008			Sample: 1991 2008		
Method	df	Value	Prob.	df	Value	Prob.	df	Value	Prob.	df	Value	Prob.
Anova F-test	(2, 2248)	12.655	0.000	(4, 1119)	10.675	0.000	(3,	2.498	0.059	(3, 353)	2.365	0.071
Analysis of Variance												
Source of Variation	df	Sum of Sq.	Mean Sq.	df	Sum of Sq.	Mean Sq.	df	Sum of Sq.	Mean Sq.	df	Sum of Sq.	Mean Sq.
Between	2	470.52	235.26	4	281.74	70.44	3	34.26	11.42	3	25.42	8.47
Within	2248	41792.62	18.59	1119	7383.49	6.60	741	3387.23	4.57	353	1264.73	3.58
Total	2250	42263.14	18.78	1123	7665.23	6.83	744	3421.50	4.60	356	1290.15	3.62
Equality of **medians**	Sample: 1881 2008			Sample: 1950 2008			Sample: 1971 2008			Sample: 1991 2008		
Method	df	Value	Prob.	df	Value	Prob.	df	Value	Prob.	df	Value	Prob.
Med. Chi-square	2	22.29	0.000	4	35.61	0.000	3	13.44	0.004	3	6.76	0.080
Adj. Med. Chi-square	2	21.67	0.000	4	32.55	0.000	3	11.14	0.011	3	5.11	0.164
Kruskal-Wallis	2	26.97	0.000	4	49.99	0.000	3	10.88	0.012	3	7.77	0.051
Kruskal-Wallis (tie-adj.)	2	26.97	0.000	4	49.99	0.000	3	10.88	0.012	3	7.77	0.051
Van der Waerden	2	25.71	0.000	4	45.27	0.000	3	9.10	0.028	3	7.24	0.065
Equality of **variances**	Sample: 1881 2008			Sample: 1950 2008			Sample: 1971 2008			Sample: 1991 2008		
Method	df	Value	Prob.	df	Value	Prob.	df	Value	Prob.	df	Value	Prob.
Bartlett	2	143.47	0.000	4	34.49	0.000	3	22.05	0.000	3	5.82	0.121
Levene	(2, 2248)	36.73	0.000	(4, 1119)	5.66	0.000	(3,	2.95	0.032	(3, 353)	0.36	0.781
Brown-Forsythe	(2, 2248)	32.60	0.000	(4, 1119)	5.63	0.000	(3,	2.91	0.034	(3, 353)	0.38	0.771
Bartlett weighted standard deviation			4.31			2.57			2.14			1.89

Category statistics of debt

Debt	Sample: 1881 2008				Debt	Sample: 1950 2008				Sample: 1971 2008				Sample: 1991 2008			
	Count	Mean	Std. Dev.	Std. Err. of Mean		Count	Mean	Std. Dev.	Std. Err. of Mean	Count	Mean	Std. Dev.	Std. Err. of Mean	Count	Mean	Std. Dev.	Std. Err. of Mean
[0, 100)	1928	2.256	4.062	0.093	[0, 50)	612	3.156	2.828	0.114	353	2.080	2.372	0.126	95	1.678	2.107	0.216
[100,	300	1.023	4.582	0.265	[50, 100)	434	2.278	2.235	0.107	334	2.204	1.946	0.106	211	2.025	1.847	0.127
[200,	23	3.918	12.991	2.709	[100, 150)	64	1.790	2.198	0.275	50	1.453	1.668	0.236	43	1.358	1.700	0.259
					[150, 250)	13	1.578	1.534	0.425	8	0.993	1.162	0.411	8	0.993	1.162	0.411
All	2251	2.109	4.334	0.091	All	1124	2.718	2.613	0.078	745	2.082	2.144	0.079	357	1.829	1.904	0.101

Notes: df is degrees of freedom. Prob. is the probability. Count is the number of observation. Sum of Sq. is the sum of square. Mean Sq. is the mean of square. Std. Dev. is the standard deviation. Std. Err. of mean is the standard error of mean.

Table A.3. Instruments correlations of public debt ratio and economic growth

Instrument	1950-2008 Growth (t+5)	1950-2008 Debt (t)	1950-1970 Growth (t+5)	1950-1970 Debt (t)	1971-1990 Growth (t+5)	1971-1990 Debt (t)	1991-2008 Growth (t+5)	1991-2008 Debt (t)	1971-2008 Growth (t+5)	1971-2008 Debt (t)
GC	-0.09	0.24	-0.12	0.23	0.05	**0.27****	0.14	0.14	0.07	0.14
GC(-1)	-0.09	0.27	-0.10	0.24	0.04	**0.32****	0.16	0.15	0.07	0.17
GC(-2)	-0.09	0.29	-0.11	0.23	0.03	**0.35****	0.18	0.15	0.07	0.20
IM	0.08	**-**	0.10	-0.36	-0.09	**-0.40****	-0.04	-0.39	-0.08	**-0.43****
IM(-1)	0.10	**-**	0.13	**-0.43***	-0.07	**-0.37****	-0.03	-0.38	-0.06	**-0.41****
IM(-2)	0.12	**-**	0.15	**-0.43***	-0.04	**-0.38****	-0.02	-0.37	-0.04	**-0.41****
EX	-0.14	**0.54****	-0.19	**0.55****	0.04	**0.55*****	-0.02	0.23	0.02	**0.46****
EX(-1)	-0.16	**0.54****	-0.22	**0.57****	0.01	**0.54*****	-0.05	0.23	-0.01	**0.46****
EX(-2)	-0.18	**0.54****	-0.25	**0.57****	-0.03	**0.54*****	-0.06	0.23	-0.03	**0.46****
GCF	-0.08	-0.11	-0.15	-0.02	0.00	-0.19	0.04	0.08	0.00	-0.19
GCF(-1)	-0.07	-0.08	-0.13	0.02	0.04	-0.21	0.06	0.08	0.04	-0.20
GCF(-2)	-0.09	-0.05	-0.13	0.04	0.03	-0.21	0.02	0.06	0.02	-0.18
ODR	**-0.23***	0.32	-0.17	-0.17	-0.01	0.02	-0.08	0.32	-0.02	0.28
ODR(-1)	**-0.23***	0.32	-0.18	-0.15	0.00	0.02	-0.07	0.30	-0.02	0.28
ODR(-2)	**-0.23***	0.33	-0.18	-0.13	0.00	0.04	-0.07	0.27	-0.01	0.27

Note: Significant at 1% (***), 5% (**) and 10% (*).

Table A.4. Fixed effects generated across countries and periods

Fixed effects for the quadratic form specification: the sample of 20 OECD countries					
Country	1950-2008	1950-1970	1971-2008	1971-1990	1991-2008
Australia	-1.78	-2.57	0.39	-0.65	15.10
Austria	-0.56	3.74	0.08	-0.05	-1.78
Belgium	2.79	-4.60	1.29	1.98	-4.19
Canada	0.45	-5.51	-0.50	-0.43	-5.90
Denmark	-1.22	2.08	-0.09	0.00	-0.93
France	-1.18	0.86	-0.31	-0.53	0.68
Germany	-1.44	2.14	-0.48	-0.47	0.21
Greece	1.04	4.70	0.59	-0.46	-4.92
Ireland	1.11	-3.46	1.13	2.14	1.11
Italy	1.46	0.29	0.19	1.00	-6.57
Japan	2.03	8.80	0.85	0.88	-2.14
Netherlands	0.40	-4.78	-0.50	-0.27	-1.92
New Zealand	-1.10	-6.50	-1.17	-1.10	4.62
Norway	-0.84	0.45	0.43	0.49	6.82
Portugal	-0.22	4.00	0.25	0.65	0.79
Spain	-0.12	4.60	0.37	0.00	0.40
Sweden	-0.57	0.88	-0.46	-0.99	-2.77
Swiss	-2.03	-0.73	-1.18	-1.45	0.01
United Kingdom	2.05	1.61	-0.25	-0.19	3.27
United States	-0.78	-5.06	-0.55	-0.37	-1.83
Fixed effects for the quadratic form specification: the sample of 10 Euro countries					
Austria	-0.80	8.13	-0.26	-0.23	-0.27
Belgium	1.78	-12.25	1.44	1.02	-6.01
France	-1.35	1.05	-0.77	-0.29	5.35
Germany	-1.53	4.48	-0.87	-0.58	4.48
Greece	0.60	7.66	0.41	-0.71	-7.76
Ireland	1.22	-9.45	0.97	1.13	4.67
Italy	0.99	-1.88	0.24	-0.12	-9.32
Netherlands	0.44	-12.01	-0.70	-1.10	-0.50
Portugal	-0.35	7.03	-0.13	0.43	5.32
Spain	-0.40	7.59	-0.08	0.27	4.00

Table A.5. Sensitivity of debt thresholds to government expenses and government effectiveness.

	1951-1970	1971-1990	1991-2008	1971-2008	1951-2008
High Government effectiveness (13 countries): Finland, Denmark, Sweden, Norway, Netherlands, Canada, New Zealand, Austria, Australia, United Kingdom, Belgium, Germany and United States.					
Model Without Fixed Effect	NS	NS	80.7***	73.8***	61.2***[a]
Model With Fixed Effect	114.7**	NS	95.4*	60.8**	92.8***[a]
Moderate Government effectiveness (7 countries): Ireland, France, Japan, Spain, Portugal, Greece and Italy.					
Model Without Fixed Effect	40.3**	41.6**	NS	49.3*	NS
Model With Fixed Effect	NS	NS	102.5***	NS	NS
High Government Final Consumption as % of GDP (9 countries): Sweden, Denmark, Netherlands, France, Canada, Belgium, Germany, United Kingdom and Norway.					
Model Without Fixed Effect	NS	NS	NS	72.2*	61.4***[a]
Model With Fixed Effect	NS	NS	NS	52.3**	101.4***[a]
Moderate Government Final Consumption as % of GDP (11 countries): New Zealand, Italy, Ireland, Austria, Australia, United States, Greece, Portugal, Japan, Spain and Switzerland.					
Model Without Fixed Effect	60.7**[a]	NS	NS	67.5***	NS
Model With Fixed Effect	NS	NS	91.5***	58.4*	NS
High Military Expenditures as % of GDP (9 countries): Unites States, United Kingdom, France, Greece, Portugal, Norway, Netherlands, Sweden and Australia.					
Model Without Fixed Effect	NS	NS	NS	45.2**	122.0*[a]
Model With Fixed Effect	NS	NS	65.3**	51.4**	118.9***[a]
Moderate Military Expenditures as % of GDP (11 countries): Belgium, Germany, Italy, New Zealand, Denmark, Spain, Canada, Switzerland, Ireland, Austria and Japan.					
Model Without Fixed Effect	46.0*[a]	NS	89.3**	NS	51.1***
Model With Fixed Effect	NS	NS	114.2**	NS	NS
High Government Transfers and Subsidies as % of GDP (7 countries): Belgium, Switzerland, Netherlands, Canada, Sweden, United States and Japan.					
Model Without Fixed Effect	53.4***	73.0*	93.0***	81.5***	NS
Model With Fixed Effect	NS	NS	NS	71.6***	NS
Moderate Government Transfers and Subsidies as % of GDP (13 countries):					
	1951-1970	1971-1990	1991-2008	1971-2008	1951-2008
Model Without Fixed Effect	NS	NS	NS	NS	NS
Model With Fixed Effect	126.7***	31.2*	NS	NS	NS

Notes: significant at 1% (***), 5% (**) and 10% (*). NS is Non-Significant. *a*: The form of Growth-debt link is convex meaning that the debt affects negatively growth for debt ratios below this threshold and positively above it.

Table A.6. Tests for cross-section dependence for the panel fixed effect model (Least Squares method)

Residual Cross-Section Dependence Test for the 20 OECD countries sample										
Degrees of freedom = 190	1950-2008		1950-1970		1971-2008		1971-1990		1991-2008	
Test	Stat.	Prob.	Stat.	Prob.	Stat.	Prob.	Stat.	Prob.	Stat.	Prob.
Breusch-Pagan LM	1362.7	0.000	474.0	0.000	855.2	0.000	539.0	0.000	671.4	0.000
Pesaran scaled LM	60.2	0.000	14.6	0.000	34.1	0.000	17.9	0.000	24.7	0.000
Bias-corrected scaled LM	60.0	0.000	14.1	0.000	33.8	0.000	17.4	0.000	23.9	0.000
Pesaran CD	32.3	0.000	15.9	0.000	23.6	0.000	17.2	0.000	22.2	0.000
Residual Cross-Section Dependence Test for the 10 Euro countries sample										
Degrees of freedom = 45	1950-2008		1950-1970		1971-2008		1971-1990		1991-2008	
Test	Stat.	Prob.	Stat.	Prob.	Stat.	Prob.	Stat.	Prob.	Stat.	Prob.
Breusch-Pagan LM	450.3	0.000	145.6	0.000	320.8	0.000	204.9	0.000	211.0	0.000
Pesaran scaled LM	42.7	0.000	10.6	0.000	29.1	0.000	16.9	0.000	17.5	0.000
Bias-corrected scaled LM	42.6	0.000	10.4	0.000	28.9	0.000	16.6	0.000	17.1	0.000
Pesaran CD	18.6	0.000	8.6	0.000	16.6	0.000	13.3	0.000	12.8	0.000

Notes: Null hypothesis: No cross-section dependence (correlation) in residuals. Test employs centered correlations computed from pairwise samples

Appendix B. Individual scatter plots for public debt and economic growth

Figure B.1. 5-year lead economic growth and current public debt, 1880 and 2008

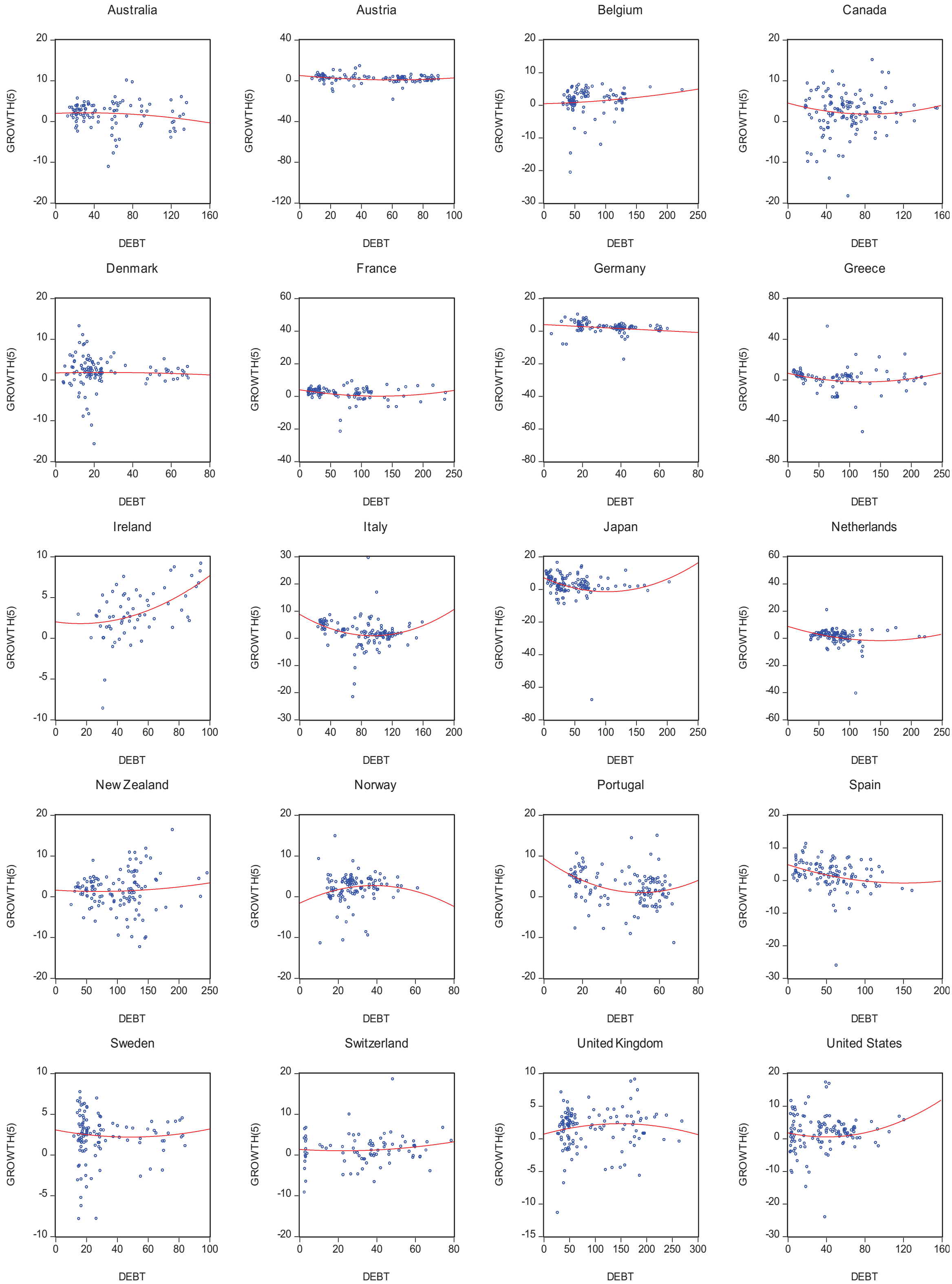


Figure B.2. 5-year lead economic growth and current public debt, 1880 and 1913

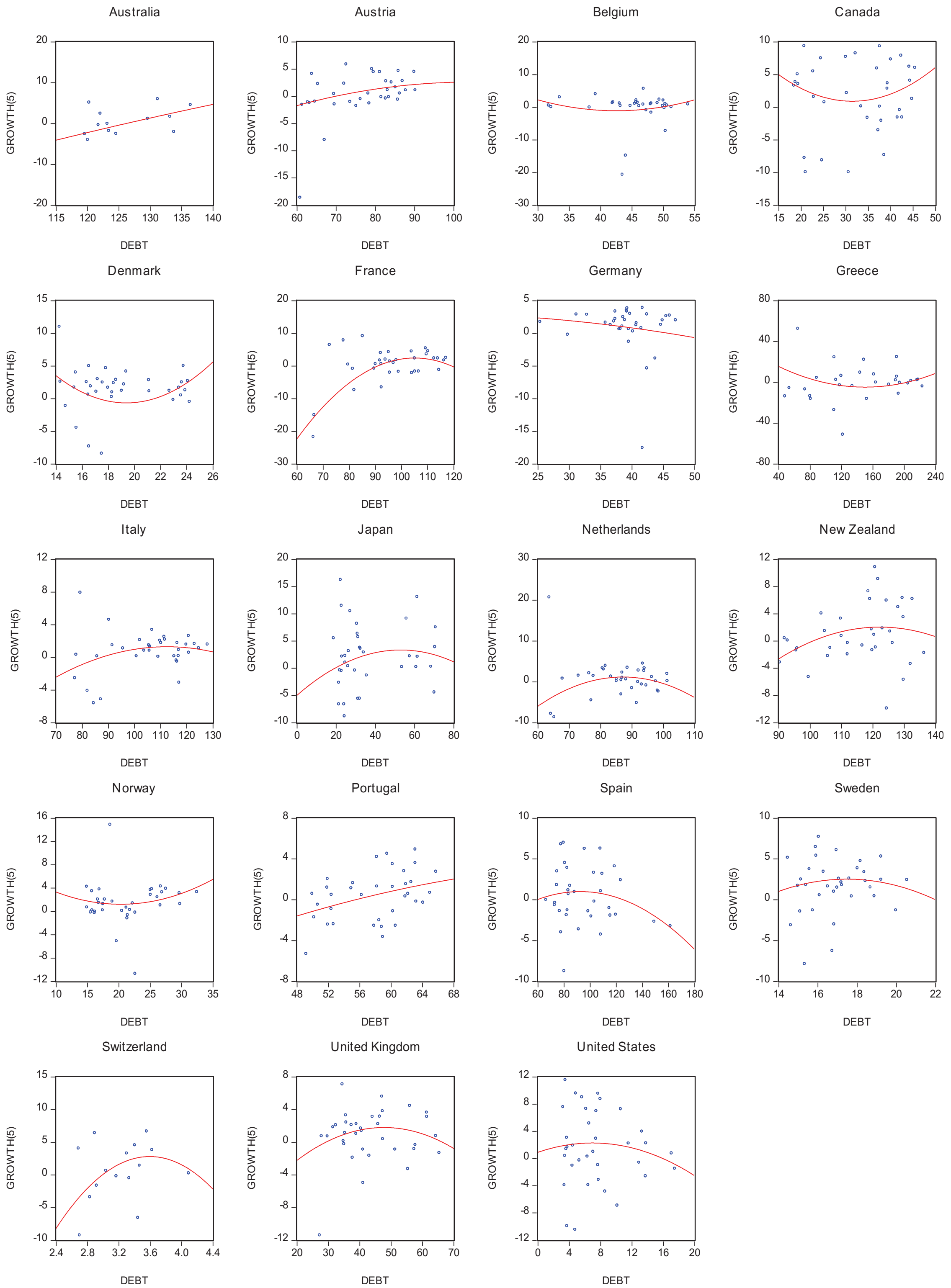


Figure B.3. 5-year lead economic growth and current public debt, 1914 and 1945

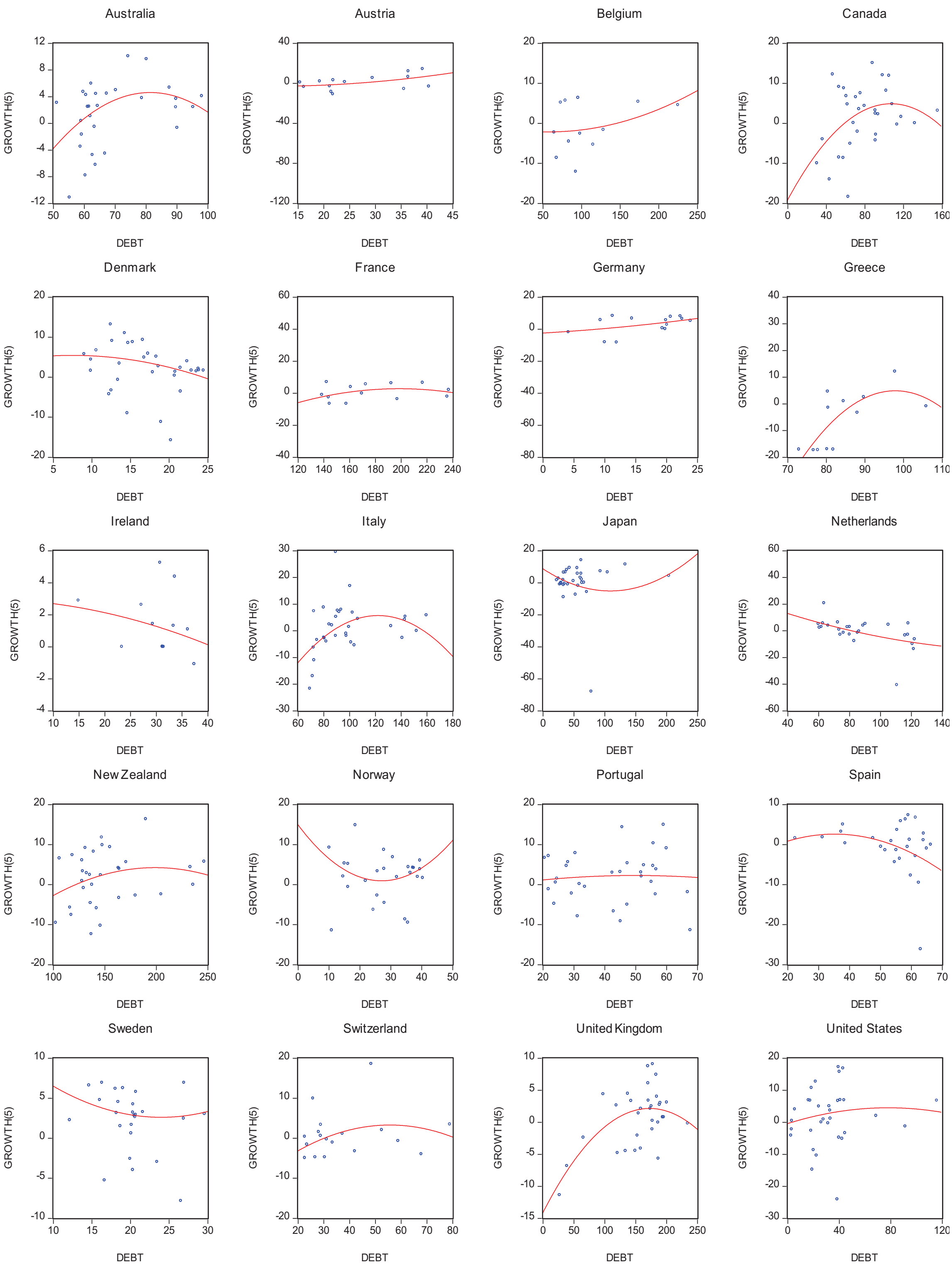


Figure B.4. 5-year lead economic growth and current public debt, 1946 and 1970

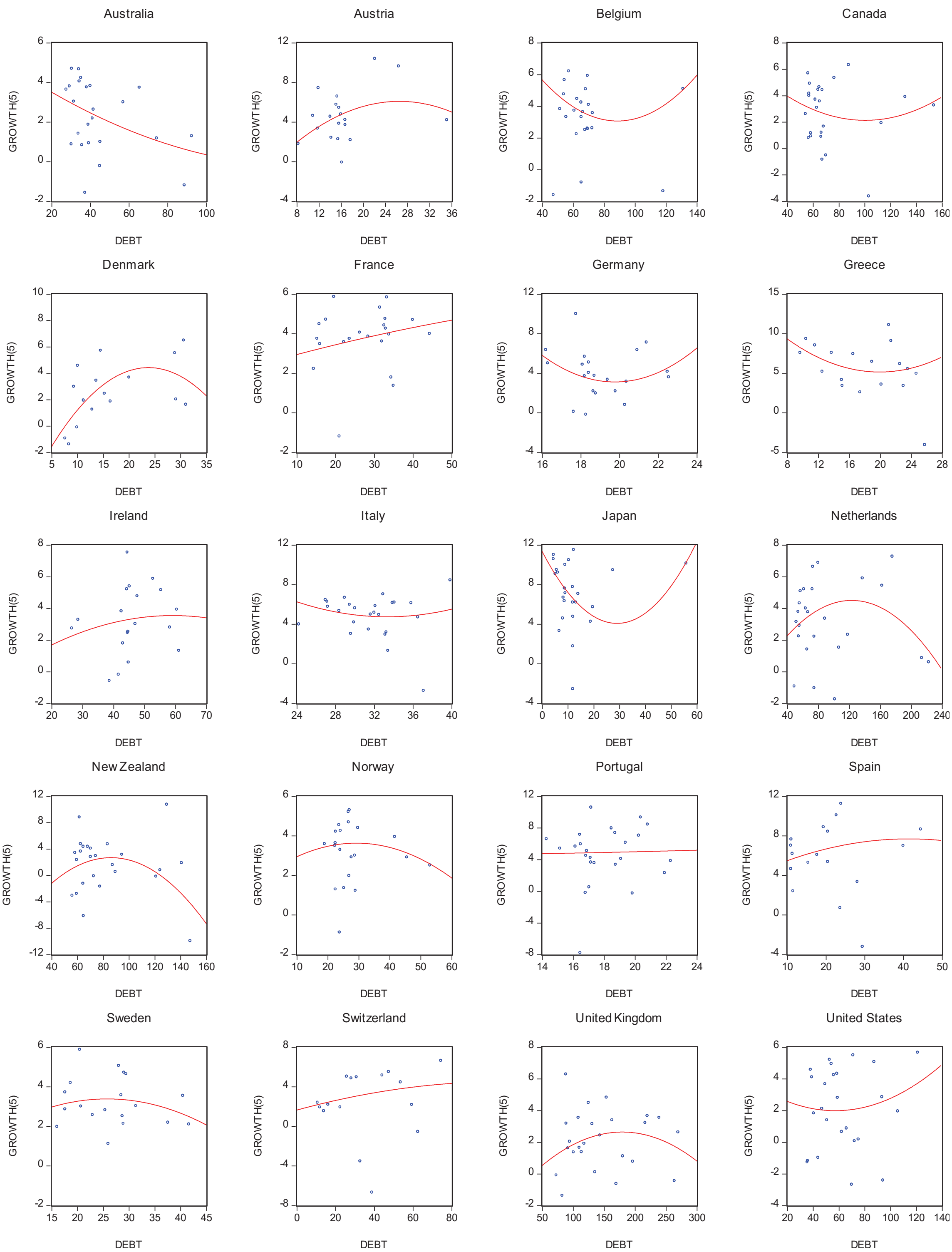


Figure B.5. 5-year lead economic growth and current public debt, 1971 and 1990

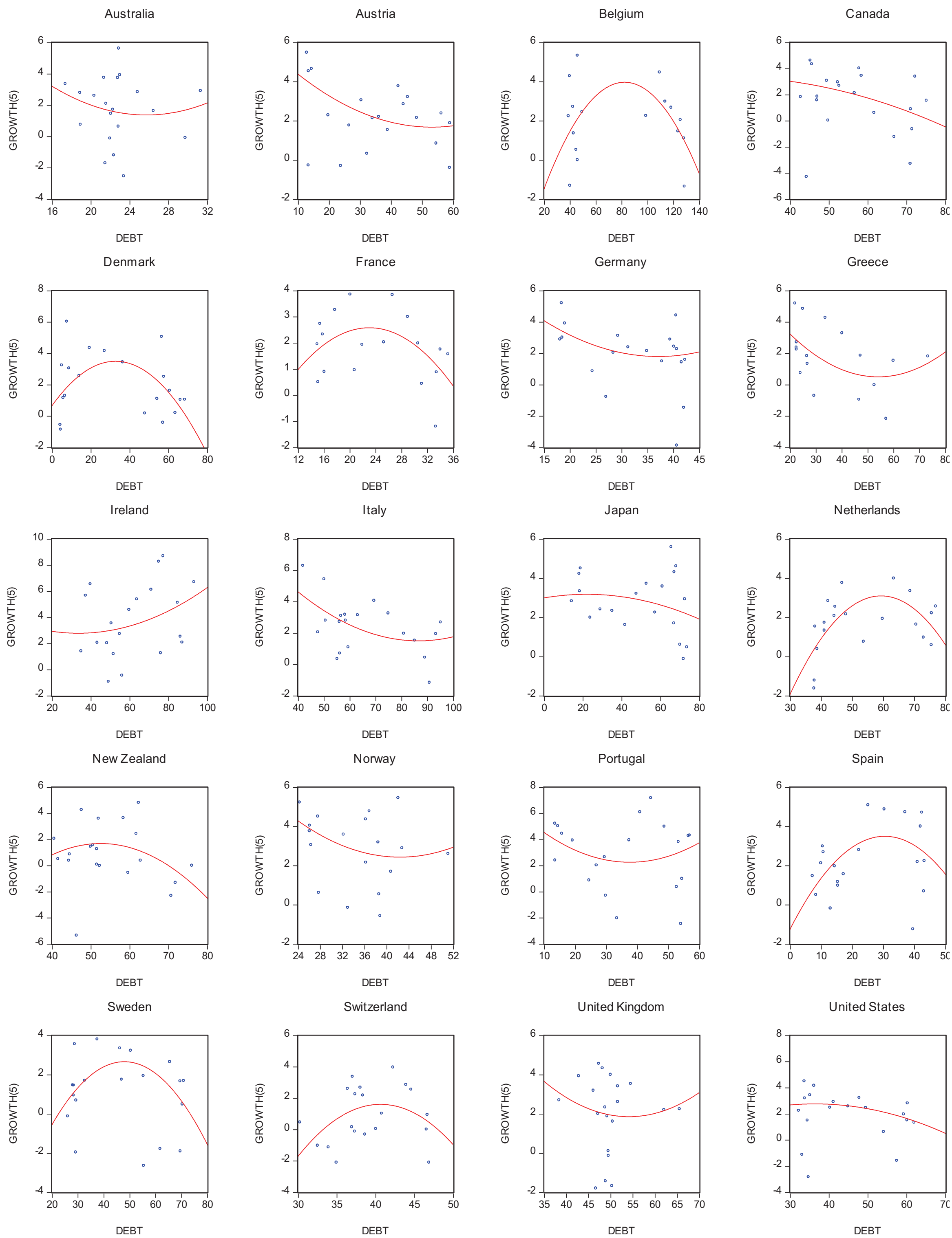


Figure B.6. 5-year lead economic growth and current public debt, 1991 and 2008

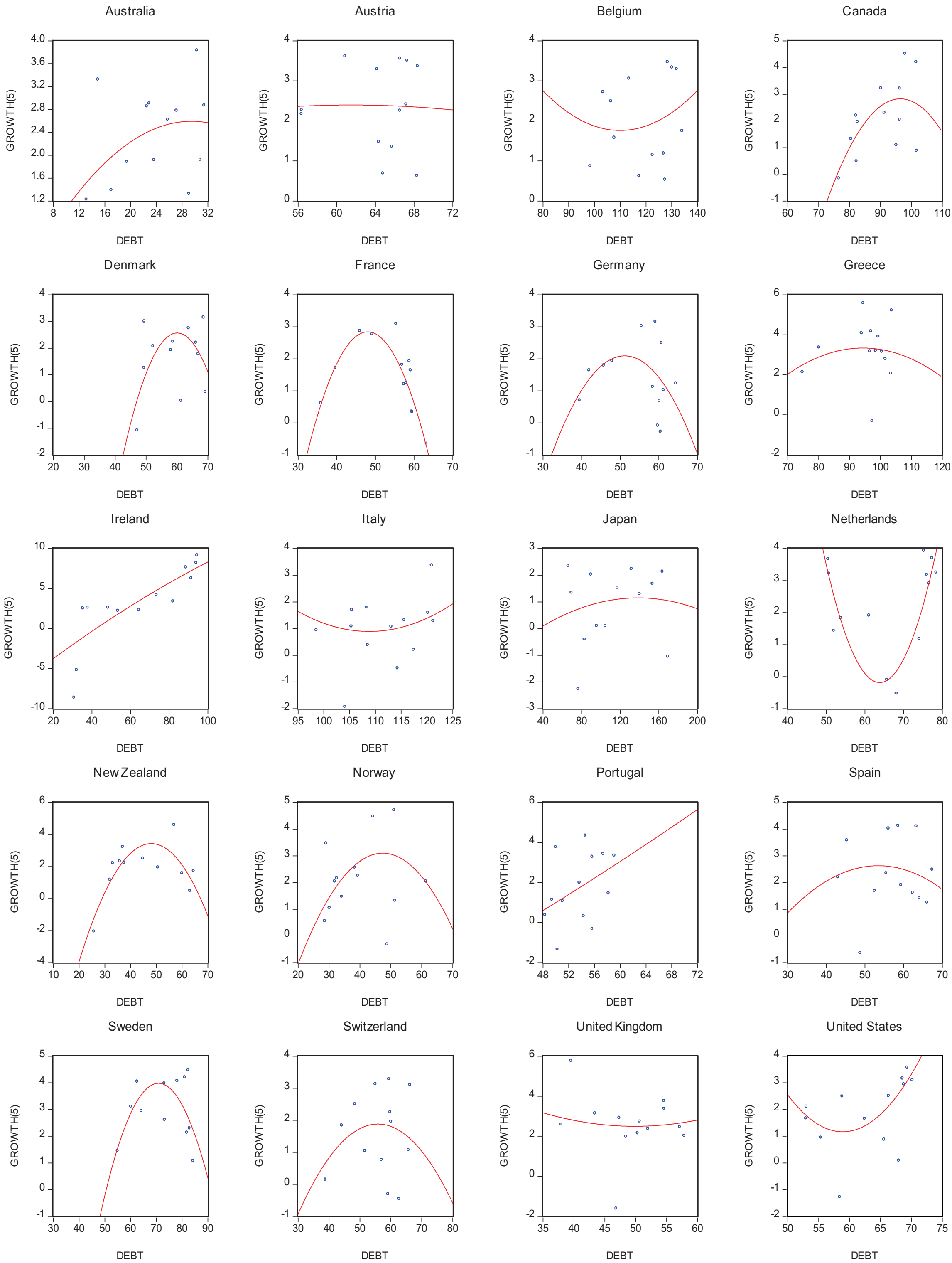


Figure B.7. 5-year lead economic growth and current public debt, 1971 and 2008

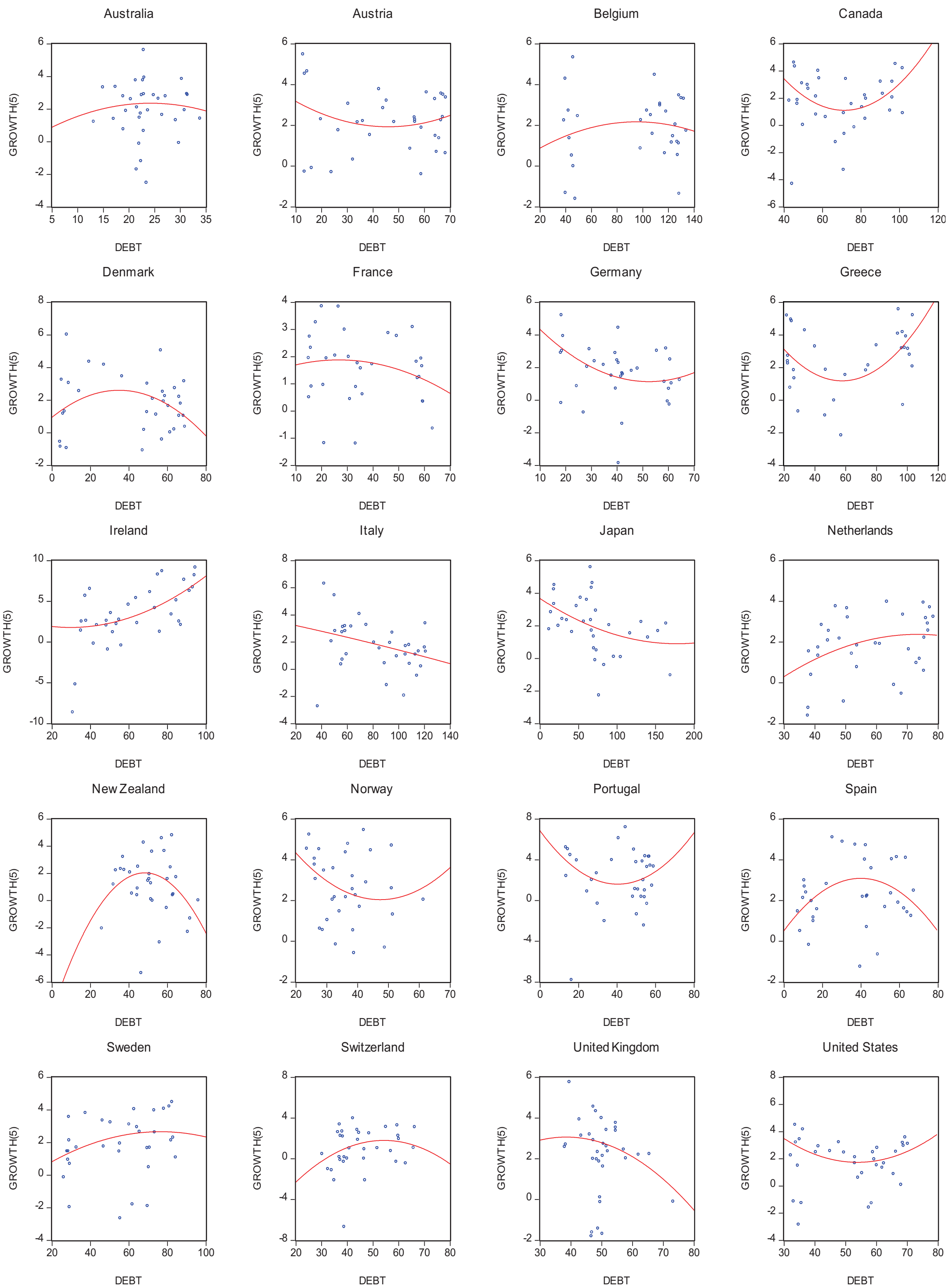


Figure B.8. 5-year lead economic growth and current public debt, 1946 and 2008

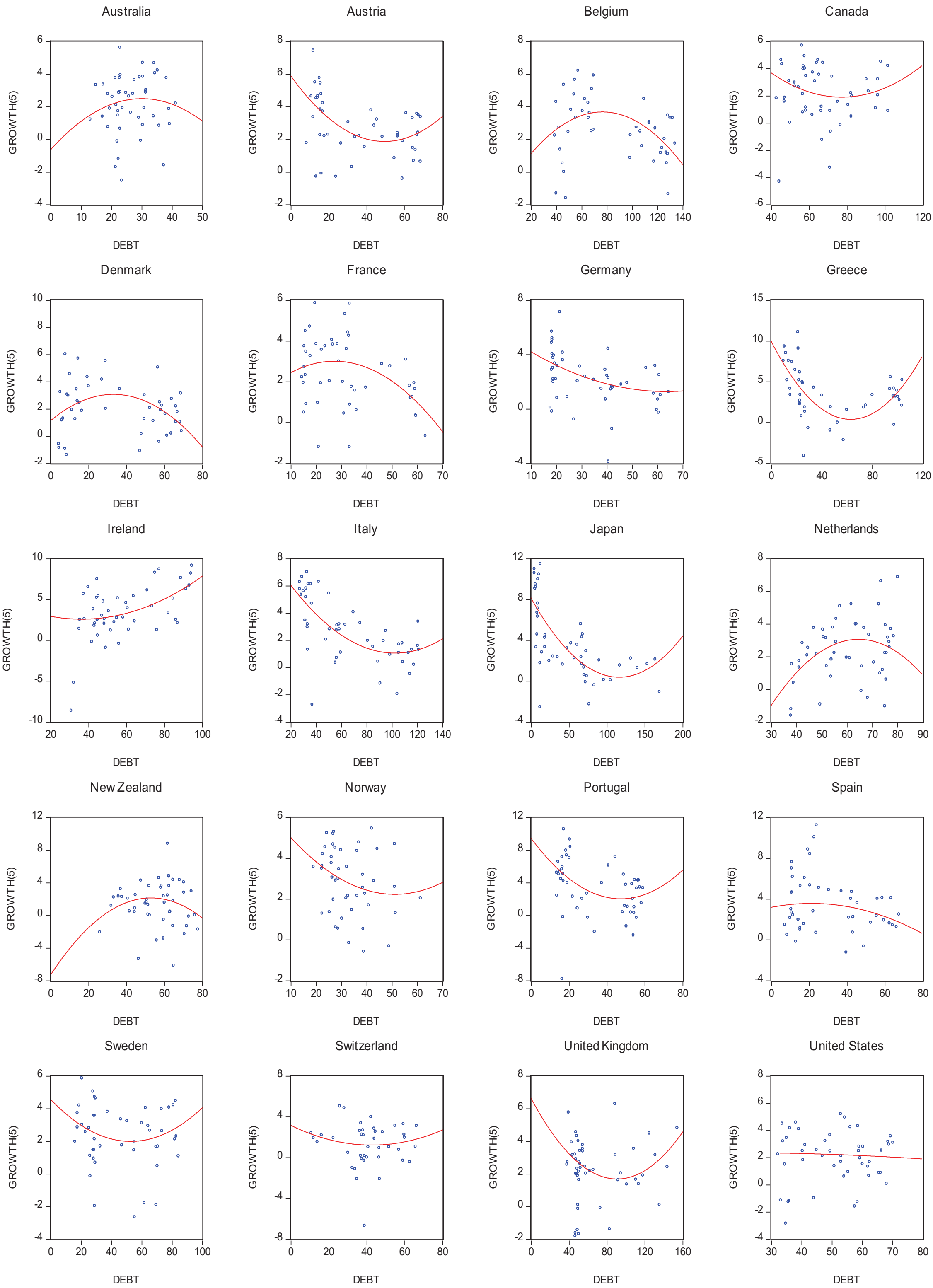
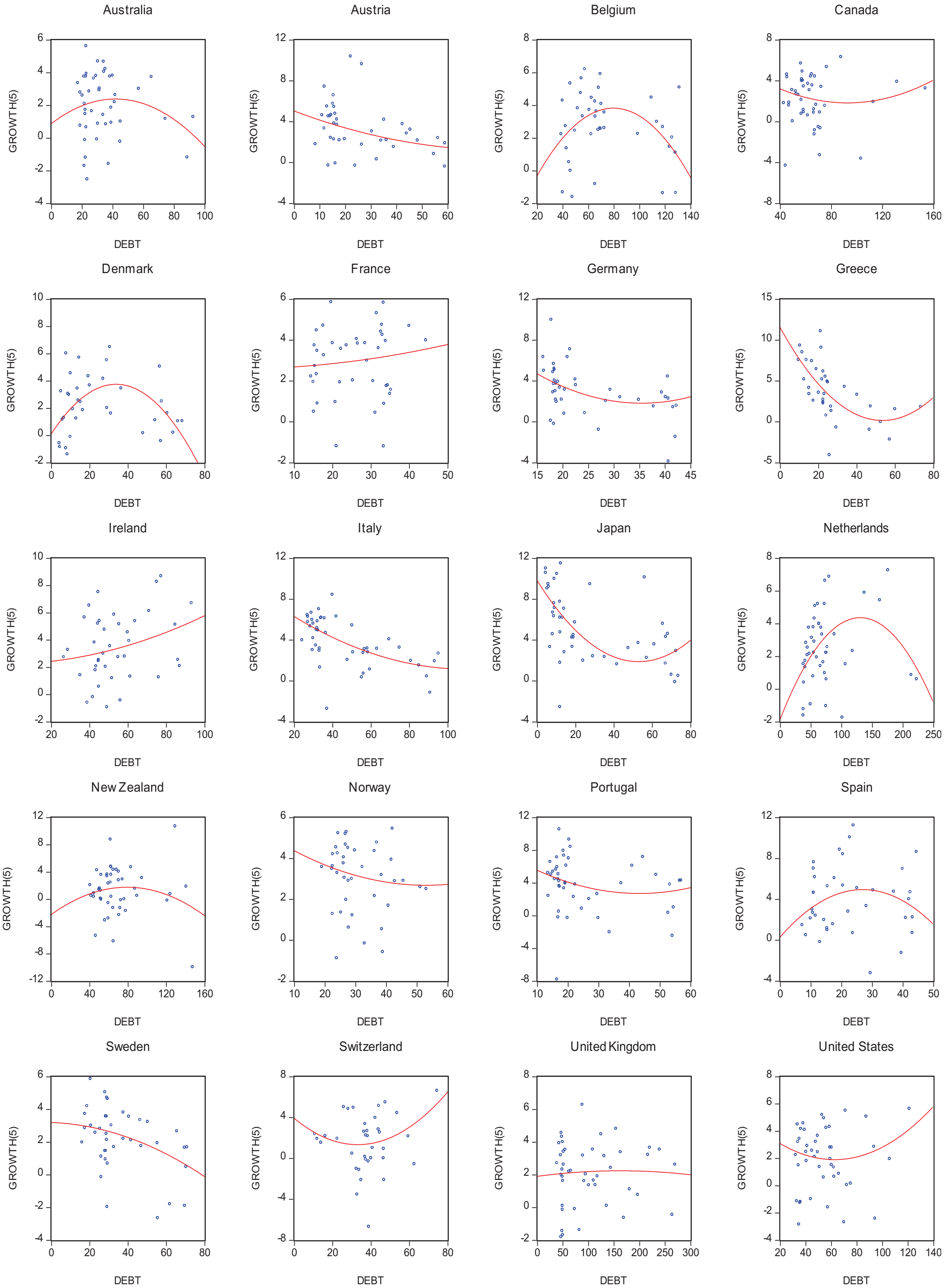


Figure B.9. 5-year lead economic growth and current public debt, 1946 and 1990



Appendix C: Hansen (2017) algorithms for the regression kink model

Algorithm 1: Testing for a Regression Kink with an Unknown Threshold.

1. Generate n *iid* draws u_t from the *N*(0,1) distribution.
2. Set $y_t^* = \widetilde{e_t} u_t$ where $\widetilde{e_t}$ are the OLS residuals from the fitted linear model (8).
3. Estimate the linear regression model (8) and the regression kink model (2), and compute the error variance estimates $\tilde{\sigma}^{*2}$ and $\hat{\sigma}^{*2}$ and the F statistic calculated as: $T_n^* = \frac{n(\tilde{\sigma}^{*2} - \hat{\sigma}^{*2})}{\hat{\sigma}^{*2}}$.
4. Repeat this *B* times to obtain a sample of simulated *F* statistic $\{T_n^*(b), b = 1..B\}$.
5. Compute the *p*-value as the percentage of simulated F statistics, which exceed the actual value: $p_n = \frac{1}{B}\sum_{b=1}^{B} 1(T_n^*(b) \geq F_n)$,
6. Compute the level α critical value c_α as the empirical $(1 - \alpha)$ of the simulated F statistics $\{T_n^*(i), i = 1..B\}$.
7. Reject $H_{0(\beta_1 = \beta_2)}$ in favor of $H_{1\,(\beta_1 \neq \beta_2)}$ at significance α if $p_n < \alpha$, or equivalently if $T_n > c_\alpha$.

   The number of bootstrap replications *B* should set sufficiently large to ensure accuracy of the p-value. We keep the number *B*=10.000 in our case as reported by Hansen (2017). We also use $\Gamma = [\min(d_t), \max(d_t)]$ for each country and a grid search with the increments of 1. The number of grid points is then: $\lambda = \max(d_t) - \min(d_t) + 1$.

Algorithm 2: Wild Bootstrap confidence intervals for parameters

1. Generate n *iid* draws u_t from the *N*(0,1) distribution.
2. Set $e_t^* = \widehat{e_t} u_t$ where $\widehat{e_t}$ are the OLS residuals from the fitted regression kink model (2).
3. Set $g_t^* = \hat{\beta}' x_t(\hat{\gamma}) + e_t^*$, where $(\hat{\beta}, \hat{\gamma})$ are the Least-Square estimates.
4. Using the observations $(g_t^*, g_{t-1}, d_{t-1})$, estimate the regression kink model (2), parameter estimates $(\widehat{\beta^*}, \widehat{\gamma^*})$ and $\hat{\sigma}^{*2} = \frac{1}{n}\sum_{t=1}^{n} \hat{e}_t^{*2}$ where $\hat{e}_t^* = g_t^* - \hat{\beta}^{*'} x_t(\hat{\gamma}^*)$.
5. Calculate the F-statistic for γ: $F_n^*(\hat{\gamma}) = \frac{n(\hat{\sigma}^{*2}(\hat{\gamma}) - \hat{\sigma}^{*2})}{\hat{\sigma}^{*2}}$; where $\hat{\sigma}^{*2} = \frac{1}{n}\sum_{t=1}^{n} \hat{e}_t^{*2}(\hat{\gamma})$ and $\hat{e}_t^*(\hat{\gamma}) = g_t^* - \hat{\beta}^{*'}(\hat{\gamma}) x_t(\hat{\gamma})$.
6. Repeat this *B* times to obtain a sample of simulated coefficient estimates $(\widehat{\beta^*}, \widehat{\gamma^*})$ and *F* statistics F_n^*.
7. Create $(1 - \alpha)$ bootstrap confidence intervals for the coefficients $\beta = (\beta_1, \beta_2, \beta_3)$ by the symmetric percentile method: the coefficients plus and minus the $(1 - \alpha)$ quantile of the absolute centered estimate bootstrap: for each coefficient β_i; $i = 1..3$, the interval is $\hat{\beta}_i \pm q_1^*$ where q_1^* is the $(1 - \alpha)$ quintile of $|\beta_i^* - \hat{\beta}_i|$.
8. Calculate the $(1 - \alpha)$ quantile $c_{1-\alpha}^*$ of the simulated *F* statistics F_n^*.
9. Create $(1 - \alpha)$ bootstrap confidence interval for γ as the set of γ for which the empirical *F* statistics $F_n(\gamma)$ are smaller than the bootstrap critical value $c_{1-\alpha}^*$: $C_\gamma^* = \{\gamma: F_n(\gamma) \leq c_{1-\alpha}^*\}$